Arc Length, Angular Velocity & Linear Velocity Name: Key Period: ____

Find the length of the arc , *L*, of a circle with radius, *r*, intercepted by a central angle, θ. Express arc length (*L*) in terms of π. (*Round your answer to the nearest hundredth.*)

	Radius, *r*	Central Angle, θ	
1	12 inches	θ = 45°	$\frac{\pi}{4}$
2	16 inches	θ = 75°	$\frac{5\pi}{12}$
3	8 feet	θ = 225°	$\frac{5\pi}{4}$
4	24 yards	θ = 320°	$\frac{16\pi}{9}$

$S = r\theta$

1. $S = 12\text{ in} \cdot \frac{\pi}{4} = 3\pi$ in.
2. $S = 16\text{ in} \cdot \frac{5\pi}{12} = \frac{80\pi}{12}$ in $\approx 6.67\pi$ in
3. $S = 8\text{ ft} \cdot \frac{5\pi}{4} = 10\pi$ ft
4. $S = 24\text{yd} \cdot \frac{16\pi}{9} = \frac{128\pi}{3}$ yds $\approx 42.67\pi$ yds

Find the positive radian measure of the angle that the second hand of a clock moves through in the given time.

1 sec = 6°

5. 55 seconds

$55 \cdot 6° = 330°$

$\frac{11\pi}{6}$

6. 35 seconds

$35 \cdot 6° = 210°$

$\frac{7\pi}{6}$

7. 3 minutes & 40 seconds

220 sec

$220 \cdot 6° = 1320$

$\frac{22\pi}{3}$

8. 4 minutes & 25 seconds

265 sec

$265 \cdot 6° = 1590°$

$\frac{53\pi}{6}$

Solve the problems.

9. **The minute hand of a clock is 9 inches long and moves from 12 to 2. How far does the tip of the minute hand move? Express you answer in terms of π and then round your answer to the nearest hundredth.**

1 min = 6°
5 min = 30°

$\theta = 60° = \frac{\pi}{3}$

$r = 9$ in

$S = r\theta = 9 \cdot \frac{\pi}{3} = 3\pi \approx 9.42$ in

10. The minute hand of a clock is 6 inches long and moves from 11 to 4. How far does the tip of the minute hand move? Express you answer in terms of π and then round your answer to the nearest hundredth.

1 min = 6°
5 min = 30°
r = 6 in

$\theta = 150° = \frac{5\pi}{6}$ → $S = r\theta$ → $S = 6\text{in}\left(\frac{5\pi}{6}\right) = 5\pi\text{in}$
$S \approx 15.71\text{in}$

Use the fact that the earth is a sphere with a radius approximately equal to 4000 miles to solve problems 11-13.

11. If two points A and B are 8000 miles apart, express and θ in radians and degrees.

$S = r\theta$
$8000 = 4000\theta$
$\theta = 2$ radians or $2 \cdot \frac{180}{\pi} \approx 114.6°$

12. If θ = 10°, find the distance between A and B to the nearest mile.

$\theta = 10° = \frac{\pi}{18}$

$S = \frac{4000}{1} \cdot \frac{\pi}{18} = \frac{4000\pi}{18} = 698.13\text{ mi} \approx 698\text{ miles}$

13. If the angular speed of the earth is $\frac{\pi}{12}$ radians per hour. The Equator lies on a circle of the radius approximately 4000 miles. Find the linear velocity of a point on the Equator in miles per hour.

$\omega = \frac{\pi}{12}$ $r = 4000$

$V = r\omega = \frac{4000\text{ mi}}{1} \cdot \frac{\pi}{12} = \frac{1000\pi}{3} \approx 1047.20\text{ mph}$

14. A water wheel has a radius of 12 feet. The wheel is rotating at 20 revolutions per minute. Find the linear speed, in feet per minute.

$r = 12\text{ft}$
$\omega = \frac{20\text{rev}}{\text{min}}$
$V = \frac{?\text{ ft}}{\text{min}}$

$V = r\omega = \frac{12\text{ft}}{1} \cdot \frac{20\text{rev}}{\text{min}} \cdot \frac{2\pi}{\text{rev}}$
$V = 1507.96\text{ ft/min} \approx 1508\text{ft/min}$

15. The second hand of a clock is 10.2 centimeters long. Find the linear speed of the tip of the second hand as it passes around clock face.

$r = 10.2\,cm$

$1\,rev = 60\,sec$

$\theta = 2\pi$

$\omega = \frac{2\pi}{60\,sec}$

$V = r\theta$

$V = \frac{10.2\,cm}{1} \cdot \frac{2\pi}{60} = 1.068\,cm/sec$

$\approx 1.07\,cm/sec$

16. A lawn roller with a 10 inch radius rotates 1.2 revolutions per second.

a. Find the angular speed per of the roller per second.

$\omega = \frac{\theta}{t} = \frac{1.2\,rev}{s} \cdot \frac{2\pi\,rad}{rev} = \frac{2.4\pi\,rad}{sec}$

b. Find the linear speed of the roller in miles per hour.

$V = r\omega = \frac{10\,in}{1} \cdot \frac{2.4\pi\,rad}{sec} \cdot \frac{60\,sec}{min} \cdot \frac{60\,min}{hr} \cdot \frac{1\,ft}{12\,in} \cdot \frac{1\,mi}{5280\,ft}$

$V = 4.28\,mph$