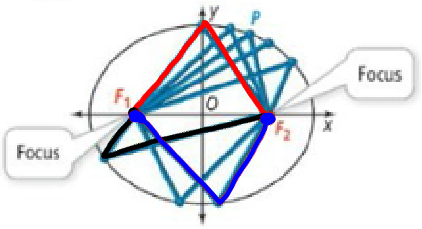
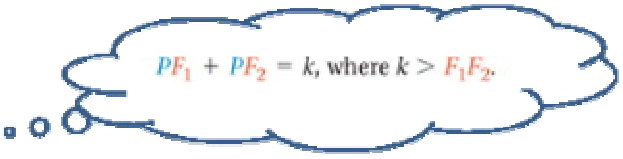
Ellipses

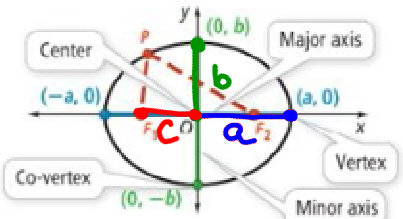
Graph



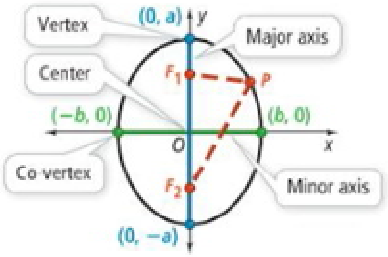


Ellipse: A set of all points *P* in a plane such that the sum of the distances from *P* to two fixed points F_1 and F_2 is a constant .

Foci of an Ellipse: The two fixed points. (The singular of *foci* is focus.)



Horizontal Ellipse



Vertical Ellipse

Major Axis: The segment that contains the two foci and has end points on the ellipse. Always the longest axis.

Center of the ellipse: The midpoint of both axes. (Where they intersect.)

Minor Axis: The segment perpendicular to the Major Axis. Goes through the center, with points on the ellipse. Always the shortest axis.

Vertices: The endpoints of the Major Axis.

Co-vertices: The endpoints of the Minor Axis.

The standard form of the equation of an ellipse with center (h,k) is:

$$\frac{(x-h)^2}{a^2}+\frac{(y-k)^2}{b^2}=1$$

Key Concept Properties of Ellipses with Center (0, 0)

	Horizontal Ellipses	Vertical Ellipses
Standard Equation	$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1, a>b>0$	$\frac{x^2}{b^2}+\frac{y^2}{a^2}=1, a>b>0$
Major Axis	horizontal	vertical
Vertices	$(\pm a, 0)$	$(0, \pm a)$
Co-vertices	$(0, \pm b)$	$(\pm b, 0)$
Foci	$(\pm c, 0)$ on x-axis	$(0, \pm c)$ on y-axis

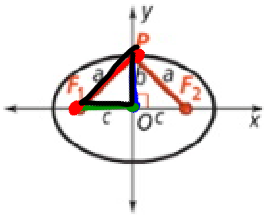
The length of the major axis is $2a$ and the length of the minor axis is $2b$.
For any point P on an ellipse, $PF_1 + PF_2 = 2a$.

Key Concept Equations of Ellipses with Centers at (h, k)

Standard Form of Equation	$\frac{(x-h)^2}{a^2}+\frac{(y-k)^2}{b^2}=1$	$\frac{(y-k)^2}{a^2}+\frac{(x-h)^2}{b^2}=1$
Direction of Major Axis	horizontal	vertical
Foci	$(h \pm c, k)$	$(h, k \pm c)$

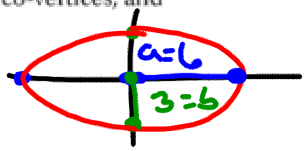
$\frac{(x-h)^2}{b^2}+\frac{(y-k)^2}{a^2}=1$

Since the co-vertex $P(0, b)$ is on the ellipse, $PF_1 + PF_2 = 2a$.
If you denote the distance from each focus to the center of the ellipse by c, then a, b, and c are the lengths of the sides of a right triangle, as shown in the ellipse at the right. Thus, the distances from the center to each vertex, to each co-vertex, and to each focus are related by the Pythagorean Theorem: $a^2 = b^2 + c^2$.



If $(\pm a, 0)$, $(0, \pm b)$, and $(\pm c, 0)$ are the vertices, the co-vertices, and the foci of an ellipse, respectively,

$$c^2 = a^2 - b^2$$



Example 11: Write the equation for an ellipse in standard form centered at the origin with vertex (-6,0) and co-vertex (0,3).

$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ $\quad a=6 \quad b=3$

$\frac{x^2}{6^2}+\frac{y^2}{3^2}=1 \Rightarrow \frac{x^2}{36}+\frac{y^2}{9}=1$

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Example 12: What are the foci of the ellipse with the equation $25x^2+9y^2=225$? Graph the foci and the ellipse.

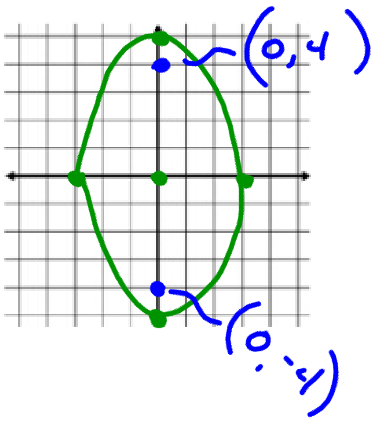
$\frac{25x^2}{225}+\frac{9y^2}{225}=\frac{225}{225}$

$\frac{x^2}{9}+\frac{y^2}{25}=1$

$a=5$

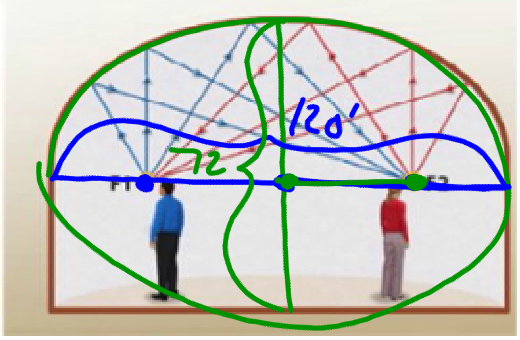
$b=3$

$c^2=a^2-b^2=25-9=16$ so $c=4$



Example 13:

Whispering Gallery A room with an elliptical ceiling (called an *ellipsoid*, since it is 3-dimensional) forms a "whispering gallery." Thanks to the reflective property of the ellipse, a whispered message at one focus can be heard clearly by someone standing across the room at the other focus. If the elliptical ceiling has a major axis of 120 feet and a minor axis of 72 feet, how far apart are the foci?



$a=60$ $b=36$

$c^2=60^2-36^2$

$c^2=3600-1296$

$c^2=2304$

$c=48$ ft $48\times2=96$ ft