A bunch of problems associated with sections 5.1 – 5.3 that in no way constitute a "review", which would indicate that some sort of assessment is imminent.

Solve the following equations for *x*. Find <u>all</u> solutions.

1. $\tan x + \sqrt{3} = 0$

$\tan x = -\sqrt{3}$

$\left.\begin{array}{l} x = \frac{2\pi}{3} \\ x = \frac{5\pi}{3} \end{array}\right\} + n\pi$

2. $2\sin x \cos x + \cos x = 0$

$\cos x(2\sin x + 1) = 0$

$\cos x = 0$ $\quad$ $2\sin x + 1 = 0$

$2\sin x = -1$

$\sin x = -\frac{1}{2} \rightarrow \left.\begin{array}{l} x = \frac{7\pi}{6} \\ x = \frac{11\pi}{6} \end{array}\right\} + 2n\pi$

$\left.\begin{array}{l} x = \frac{\pi}{2} \\ x = \frac{3\pi}{2} \end{array}\right\} + 2n\pi$

3. $4\sin^2 x = 3$

$\sin^2 x = \frac{3}{4}$

$\sin x = \pm\sqrt{\frac{3}{4}} = \pm\frac{\sqrt{3}}{2}$

$x = \left.\frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}\right\} + 2n\pi$

4. $\sin^2 x + \sin x - 2 = 0$

$(\sin x + 2)(\sin x - 1) = 0$

$\sin x + 2 = 0$ $\quad$ $\sin x - 1 = 0$

$\sin x = -2$ $\quad$ $\sin x = 1$

NO SOLUTION $\quad$ $x = \frac{\pi}{2} + 2n\pi$

5. $3\cos^2 x = \sin^2 x$

$3\cos^2 x - \sin^2 x = 0$

$3\cos^2 x - (1 - \cos^2 x) = 0$

$3\cos^2 x - 1 + \cos^2 x = 0$

$4\cos^2 x - 1 = 0$

$4\cos^2 x = 1$

$\cos^2 x = \frac{1}{4}$ $\rightarrow \cos x = \pm\frac{1}{2}$

$x = \underbrace{\frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}}_{+2n\pi}$

6. $3\cot^2 x - 1 = 0$

$3\cot^2 x = 1$

$\cot^2 x = \frac{1}{3}$

$\cot x = \pm\frac{1}{\sqrt{3}}$

$\tan x = \pm\sqrt{3}$

$x = \underbrace{\frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}}_{+2n\pi}$

7. $4\cos^2 x - 1 = 0$

$4\cos^2 x = 1$

$\cos^2 x = 1/4$

$\cos x = \pm\frac{1}{2}$

$x = \left.\begin{array}{l} \frac{\pi}{3}, \frac{2\pi}{3}, \\ \frac{4\pi}{3}, \frac{5\pi}{3} \end{array}\right\} + 2n\pi$

8. $3\sec^2 x - 4 = 0$

$3\sec^2 x = 4$

$\sec^2 x = \frac{4}{3}$

$\sec x = \pm\frac{2}{\sqrt{3}}$

$\cos x = \pm\frac{\sqrt{3}}{2}$

$x = \underbrace{\frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}}_{+2n\pi}$

9. $4\tan^2 x - 1 = \tan^2 x$

$3\tan^2 x - 1 = 0$

$3\tan^2 x = 1$

$\tan^2 x = \frac{1}{3}$

$\tan x = \pm\frac{1}{\sqrt{3}} = \pm\frac{\sqrt{3}}{3}$

$x = \left.\begin{array}{l} \frac{\pi}{6}, \frac{5\pi}{6} \\ \frac{7\pi}{6}, \frac{11\pi}{6} \end{array}\right\} + 2n\pi$

10. $\sec x \csc x = 2\csc x$

$\sec x \csc x - 2\csc x = 0$

$\csc x(\sec x - 2) = 0$

$\csc x = 0$ $\quad$ $\sec x - 2 = 0$

$\sin x = \frac{1}{0}$ $\quad$ $\sec x = 2$

NO SOLUTION $\quad$ $\cos x = \frac{1}{2}$

$x = \underbrace{\frac{\pi}{3}, \frac{5\pi}{3}}_{+2n\pi}$

11. $\sin^2 x = 3\cos^2 x$

$\sin^2 x - 3\cos^2 x = 0$

$1-\cos^2 x - 3\cos^2 x = 0$

$1 - 4\cos^2 x = 0$

$1 = 4\cos^2 x$

$\frac{1}{4} = \cos^2 x$

$\cos x = \pm\frac{1}{2}$

$\left.\begin{array}{l} x = \frac{\pi}{3}, \frac{2\pi}{3}, \\ \frac{4\pi}{3}, \frac{5\pi}{3} \end{array}\right\} + 2n\pi$

12. $2\sin^2 x = 2 + \cos x$

$2\sin^2 x - \cos x - 2 = 0$

$2(1-\cos^2 x) - \cos x - 2 = 0$

$\cancel{2} - 2\cos^2 x - \cos x - \cancel{2} = 0$

$-2\cos^2 x - \cos x = 0$

$-\cos x(2\cos x + 1) = 0$

$-\cos x = 0 \qquad 2\cos x + 1 = 0$

$\cos x = -\frac{1}{2}$

$\left.\begin{array}{l} x = \frac{\pi}{2}, \frac{3\pi}{2} \\ x = \frac{\pi}{3}, \frac{5\pi}{3} \end{array}\right\} + 2n\pi$

13. $\sec^2 x - \sec x = 2$

$\sec^2 x - \sec x - 2 = 0$

$(\sec x - 2)(\sec x + 1) = 0$

$\sec x - 2 = 0 \qquad \sec x + 1 = 0$

$\sec x = 2 \qquad \sec x = 1$

$\cos x = \frac{1}{2} \qquad \cos x = 1$

$\left.\begin{array}{l} x = \frac{\pi}{3}, \frac{5\pi}{3} \\ x = \pi \end{array}\right\} + 2n\pi$

14. $\cos^2 x + \sin x = 1$

$\cos^2 x + \sin x - 1 = 0$

$1 - \sin^2 x + \sin x - 1 = 0$

$-\sin^2 x + \sin x = 0$

$-\sin x(\sin x - 1) = 0$

$-\sin x = 0 \qquad \sin x - 1 = 0$

$\sin x = 1$

$\underbrace{x = 0, \pi}_{+2n\pi} \qquad x = \frac{\pi}{2} + 2n\pi$

15. $\tan^2 x + \sec x - 1 = 0$

$\sec^2 x - 1 + \sec x - 1 = 0$

$\sec^2 x + \sec x - 2 = 0$

$(\sec x + 2)(\sec x - 1) = 0$

$\sec x + 2 = 0 \qquad \sec x - 1 = 0$

$\sec x = -2 \qquad \sec x = 1$

$\cos x = -\frac{1}{2} \qquad \cos x = 1$

$\underbrace{x = \frac{2\pi}{3}, \frac{4\pi}{3}}_{+2n\pi}$

$x = 0 + 2n\pi$

Verify that each of the following is an identity.

16. $(1+\sin\theta)(1-\sin\theta) = \frac{1}{\sec^2\theta}$

$= 1 - \sin^2\theta$

$= \cos^2\theta$

$= \frac{1}{\sec^2\theta} = \frac{1}{\sec^2\theta}$ ✓

17. $\cos^2 x \cot^2 x = \cot^2 x - \cos^2 x$

$= \frac{\cos^2 x}{\sin^2 x} - \cos^2 x$

$= \frac{\cos^2 x - \cos^2 x \sin^2 x}{\sin^2 x}$

$= \frac{\cos^2 x(1 - \sin^2 x)}{\sin^2 x}$

$= \frac{\cos^2 x \cdot \cos^2 x}{\sin^2 x}$ ✓

$= \frac{\cos^2 x}{\sin^2 x} \cdot \cos^2 x = \cot^2 x \cdot \cos^2 x$

18. $\tan^4 w + 2\tan^2 w + 1 = \sec^4 w$

$= (\tan^2 w + 1)^2$

$= (\sec^2 w)^2$

$= \sec^4 w = \sec^4 w$ ✓

19. $\sin^2 x(\csc^2 x + \sec^2 x) = \sec^2 x$

$= \sin^2 x \csc^2 x + \sin^2 x \sec^2 x$

$= 1 + \sin^2 x \cdot \frac{1}{\cos^2 x}$

$= 1 + \frac{\sin^2 x}{\cos^2 x}$

$= 1 + \cot^2 x$

$= \sec^2 x = \sec^2 x$ ✓

20. $\dfrac{\sin x+\cos x}{1-\sin x}=\dfrac{1+\cot x}{\csc x-1}$

$$=\frac{1+\frac{\cos x}{\sin x}}{\frac{1}{\sin x}-1}$$

$$=\frac{\frac{\sin x+\cos x}{\cancel{\sin x}}}{\frac{1-\sin x}{\cancel{\sin x}}}$$

$$\frac{\sin x+\cos x}{1-\sin x}=\frac{\sin x+\cos x}{1-\sin x}\checkmark$$

21. $\dfrac{1-\tan x}{1+\tan x}=\dfrac{\cot x-1}{\cot x+1}$

WORK BOTH SIDES...

$$=\frac{1-\frac{\sin x}{\cos x}}{1+\frac{\sin x}{\cos x}}\circ\frac{\frac{\cos x}{\sin x}-1}{\frac{\cos x}{\sin y}+\cancel{1}0}$$

$$=\frac{\frac{\cos x-\sin x}{\cancel{\cos x}}}{\frac{\cos x+\sin x}{\cancel{\cos x}}}=\frac{\frac{\cos x-\sin x}{\cancel{\sin x}}}{\frac{\cos x+\sin x}{\cancel{\sin x}}}$$

$$\boxed{\frac{\cos x-\sin x}{\cos x+\sin x}=\frac{\cos x-\sin x}{\cos x+\sin x}}$$

22. $\dfrac{\sec\theta-1}{1-\cos\theta}=\sec\theta$

$$=\frac{\frac{1}{\cos\theta}-1}{1-\cos\theta}$$

$$=\frac{\frac{1-\cos\theta}{\cos\theta}}{1-\cos\theta}$$

$$=\frac{\cancel{1-\cos\theta}}{\cos\theta}\cdot\frac{1}{\cancel{1-\cos\theta}}$$

$$=\frac{1}{\cos\theta}=\sec\theta=\sec\theta\checkmark$$

23. $\dfrac{\sec^2 x-1}{\sec^2 x}=\sin^2 x$

$$=\frac{\tan^2 x}{\sec^2 x}$$

$$=\frac{\frac{\sin^2 x}{\cancel{\cos^2 x}}}{\frac{1}{\cancel{\cos^2 x}}}$$

$$=\sin^2 x=\sin^2 x$$

24. $\dfrac{\cos y}{1-\sin y}=\sec y+\tan y$

$$\frac{(1+\sin y)}{(1+\sin y)}$$

$$=\frac{(1+\sin y)(\cos y)}{1-\sin^2 y}$$

$$=\frac{(1+\sin y)\cancel{(\cos y)}}{\cos^{\cancel{2}} y}$$

$$=\frac{1+\sin y}{\cos y}$$

$$=\frac{1}{\cos y}+\frac{\sin y}{\cos y}$$

$$=\sec y+\tan y=\sec y+\tan y\checkmark$$

25. $\dfrac{\cos x}{1-\sin x}=\dfrac{1+\sin x}{\cos x}$

$$\frac{(1+\sin x)}{(1+\sin x)}$$

$$=\frac{(1+\sin x)(\cos x)}{1-\sin^2 x}$$

$$=\frac{(1+\sin x)\cancel{(\cos x)}}{\cos^{\cancel{2}} x}$$

$$=\frac{1+\sin x}{\cos x}=\frac{1+\sin x}{\cos x}\checkmark$$

26. Use a sum or difference formula to find sin 285º and tan 285º.

$\sin 285^\circ = \sin(240^\circ + 45^\circ)$

$= \sin 240^\circ \cos 45^\circ + \cos 240^\circ \sin 45^\circ$

$= -\frac{\sqrt{3}}{2} \cdot \frac{\sqrt{2}}{2} + -\frac{1}{2} \cdot \frac{\sqrt{2}}{2}$

$\boxed{= \frac{-\sqrt{6} - \sqrt{2}}{4}}$

$\tan 285^\circ = \tan(240^\circ + 45^\circ)$

$= \frac{\tan 240^\circ + \tan 45^\circ}{1 - \tan 240^\circ \tan 45^\circ}$

$= \frac{\sqrt{3} + 1}{1 - (\sqrt{3})(1)}$

$= \frac{(\sqrt{3}+1)(1+\sqrt{3})}{(1-\sqrt{3})(1+\sqrt{3})}$

$= \frac{\sqrt{3} + 1 + 3 + \sqrt{3}}{1 - 3}$

$= \frac{4 + 2\sqrt{3}}{-2} = \boxed{-2 - \sqrt{3}}$

27. Verify: $\cos\left(\frac{5\pi}{4} + x\right) = -\frac{\sqrt{2}}{2}(\cos x - \sin x)$

$= \cos\frac{5\pi}{4}\cos x - \sin\frac{5\pi}{4}\sin x$

$= -\frac{\sqrt{2}}{2}\cos x - \left(-\frac{\sqrt{2}}{2}\right)\sin x$

$= -\frac{\sqrt{2}}{2}(\cos x - \sin x)$ ✓

28. Verify: $\sin\left(\frac{\pi}{6} + x\right) = \frac{1}{2}\left(\cos x + \sqrt{3}\sin x\right)$

$= \sin\frac{\pi}{6}\cos x + \cos\frac{\pi}{6}\sin x$

$= \frac{1}{2}\cos x + \frac{\sqrt{3}}{2}\sin x$

$= \frac{1}{2}(\cos x + \sqrt{3}\sin x)$

29. Simplify: $\sin\left(\frac{3\pi}{2} + \theta\right) = \sin\frac{3\pi}{2}\cos\theta - \cos\frac{3\pi}{2}\sin\theta$

$= (-1)\cos\theta - (0)\sin\theta$

$= -\cos\theta$

Find the <u>exact values</u> of the following:

30. $\sin 330^\circ \cos 30^\circ - \cos 330^\circ \sin 30^\circ$

$= \sin(330^\circ - 30^\circ) = \sin 300^\circ = \frac{-\sqrt{3}}{2}$

OR

$-\frac{1}{2} \cdot \frac{\sqrt{3}}{2} - \frac{\sqrt{3}}{2} \cdot \frac{1}{2} = \frac{-\sqrt{3}}{4} - \frac{\sqrt{3}}{4} = -\frac{2\sqrt{3}}{4} = \frac{-\sqrt{3}}{2}$

31. $\sin\frac{\pi}{12}\cos\frac{\pi}{4} - \cos\frac{\pi}{12}\sin\frac{\pi}{4}$

$= \sin\left(\frac{\pi}{12} - \frac{\pi}{4}\right) = \sin\left(\frac{\pi}{12} - \frac{3\pi}{12}\right) = \sin\left(-\frac{2\pi}{12}\right) = \sin\left(-\frac{\pi}{6}\right) = -\frac{1}{2}$

OR

$= \sin 15^\circ \cos 45^\circ - \cos 15^\circ \sin 45^\circ = \sin(15^\circ - 45^\circ) = \sin(-30^\circ) = -\frac{1}{2}$

32. $\frac{\tan 25^\circ + \tan 110^\circ}{1 - \tan 25^\circ \tan 110^\circ} = \tan(25^\circ + 110^\circ)$

$= \tan(135^\circ) = -1$