HPC/RPC Review(2) Name_____ Period_____

Properties of Exponentials and Logarithms
and Solving Exponential and Logarithmic equations

Solve the problem.

1) There are currently 73 million cars in a certain country, increasing by 1.7% annually. How many years will it take for this country to have 91 million cars? Round to the nearest year. $(1+r)^t$

$91 = 73(1+0.017)^t$ $\quad \frac{91}{73} = 1.017^t$

$91 = 73(1.017)^t$ $\quad \log_{1.017}\left(\frac{91}{73}\right) = t \approx 13 \text{ yrs}$

2) A bacterial culture has an initial population of 10,000. If its population declines to 6000 in 4 hours, what will it be at the end of 6 hours? $(1+r)^t$

$6000 = 10000(1+r)^4$

$\frac{6}{10} = (1+r)^4$ $\quad \sqrt[4]{0.3} = 1+r$ $\quad A = 10{,}000(1-0.26)^6$

$r = \sqrt[4]{0.3} - 1$ $\quad \approx 165$

$r \approx -0.26$

3) The population of wolves in a state park after t years is modeled by the function $P(t) = \frac{800}{1 + 99e^{-0.3t}}$.

What was the initial population of wolves? $\frac{800}{1+99} = \frac{800}{100} = 8$

What is the maximum sustainable population? 800 wolves

About how many years will it take for the wolf population to reach 200?

$200 = \frac{800}{1+99e^{-0.3t}} \rightarrow 1+99e^{-0.3t} = 4 \rightarrow e^{-0.3t} = \frac{3}{99}$

$99e^{-0.3t} = 3$ $\quad -0.3t = \ln\left(\frac{1}{33}\right)$ $\quad t = \frac{\ln\left(\frac{1}{33}\right)}{-0.3}$

$t \approx 11.7 \text{ yrs}$

Use Newton's Law of Cooling, $T = C + (T_0 - C)e^{kt}$, for problems 4 & 5. T_0

4) A lasagna removed from the oven has a temperature of 430°F. It is left sitting in a room that has a temperature of 65°F. After 6 minutes, the temperature of the lasagna is 320°F. Use Newton's Law of Cooling to find a model for the temperature of the lasagna, T, after t minutes. C, t, T

$320 = 65 + (430 - 65)e^{6k}$

$255 = 365e^{6k} \rightarrow \frac{51}{73} = e^{6k}$ $\quad \frac{\ln\frac{51}{73}}{6} = k \approx -0.035$

$T = 65 + 365e^{-0.035t}$

5) A cake is removed from an oven at 325 °F and cools to 150 °F after 25 minutes in a room 68 °F. How long will it take the cake to cool to 113 °F? T_0, T, t, C

$150 = 68 + (325-68)e^{25k}$

$82 = 257e^{25k}$

$\frac{82}{257} = e^{25k}$ $\quad \ln\frac{82}{257} = 25k$

$k = \frac{\ln\frac{82}{257}}{25}$

$k \approx -0.046$

$113 = 68 + (257)e^{-0.046t}$

$45 = 257e^{-0.046t}$

$\frac{45}{257} = e^{-0.046t}$

$\ln\frac{45}{257} = -0.046t$ $\quad t = \frac{\ln\frac{45}{257}}{-0.046}$

$t \approx 38.13$ min.

$A = P(1+\frac{r}{n})^{nt}$ $n=1$

6) How long must $6000 be in a bank at 7% compounded annually to become $11,802.91? (Round to the nearest year.)

$11{,}802.91 = 6000(1+0.07)^t$

$\frac{11{,}802.91}{6000} = 1.07^t$ → $\log_{1.07}\left(\frac{11{,}802.91}{6000}\right) = t$

$t \approx 10$ yrs

$A = Pe^{rt}$

7) How long will it take for $5500 to grow to $22,600 at an interest rate of 3% if the interest is compounded continuously? Round the number of years to the nearest hundredth.

$22{,}600 = 5500\,e^{0.03t}$

$\frac{22600}{5500} = e^{0.03t}$ → $\ln\frac{22600}{5500} = 0.03t$

$t = \frac{\ln\frac{22600}{5500}}{0.03} \approx 47.4$ yrs

8) Find how long it will take for $6200 invested at 9.275% per year compounded daily to triple in value. Find the answer to the nearest year. $n = 365$

$3 = 1\left(1+\frac{0.09275}{365}\right)^{365t}$

$3 = 1.000254^{365t}$ → $\log_{1.000254} 3 = 365t$

$t = \frac{\log_{1.000254} 3}{365} \approx 11.85$ yr ≈ 12 yrs

Determine the doubling time of the investment.

9) 4.37% APR compounded quarterly

$2 = 1\left(1+\frac{0.0437}{4}\right)^{4t}$

$2 = 1.010925^{4t}$ → $\frac{\log_{1.010925} 2}{4} = t \approx 15.95$ yrs

10) 5.08% APR compounded continuously

$2 = 1e^{0.0508t}$

$\ln 2 = 0.0508t$

$t = \frac{\ln 2}{0.0508} \approx 13.64$ yrs

Solve.

11) The function $A = A_0e^{-0.0077x}$ models the amount in pounds of a particular radioactive material stored in a concrete vault, where x is the number of years since the material was put into the vault. If 700 pounds of the material are placed in the vault, how much time will need to pass for only 150 pounds to remain?

$A_0 = 700$

$A = 150$

$150 = 700e^{-0.0077x}$

$\frac{150}{700} = e^{-0.0077x}$

$\ln\left(\frac{150}{700}\right) = -0.0077x$

$\frac{\ln\left(\frac{150}{700}\right)}{-0.0077} = x \approx 200$ yrs