

Verifying Trigonometric Identities

Name: _____ Date: _____

1. $\frac{\sec^2\theta - 1}{\sec^2\theta} = \sin^2\theta$

$$\frac{\tan^2\theta}{\sec^2\theta} = \frac{\sin^2\theta}{\cos^2\theta} \cdot \frac{1}{\sec^2\theta} = \frac{\sin^2\theta}{\cos^2\theta} \cdot \frac{\cos^2\theta}{1} = \sin^2\theta \checkmark$$

6. $\frac{1}{1-\sin\alpha} + \frac{1}{1+\sin\alpha} = 2\sec^2\alpha$

$$\frac{1+\sin\alpha + 1-\sin\alpha}{1-\sin^2\alpha} = \frac{2}{1-\sin^2\alpha} = \frac{2}{\cos^2\alpha} = 2\sec^2\alpha \checkmark$$

2. $\sec\beta \cos\beta = 1$

$$\frac{1}{\cos\beta} \cdot \frac{\cos\beta}{1} = \frac{\cos\beta}{\cos\beta} = 1 \checkmark$$

7. $\tan^2\beta \sin^2\beta = \tan^2\beta - \sin^2\beta$

$$\begin{aligned} \tan^2\beta (1 - \cos^2\beta) &= \\ \tan^2\beta - \tan^2\beta \cos^2\beta &= \\ \tan^2\beta - \frac{\sin^2\beta}{\cos^2\beta} \cdot \frac{\cos^2\beta}{1} &= \\ \tan^2\beta - \sin^2\beta &\checkmark \end{aligned}$$

3. $2 - \sec^2\theta = 1 - \tan^2\theta$

$$\begin{aligned} 2 - (1 + \tan^2\theta) &= \\ 2 - 1 - \tan^2\theta &= \\ 1 - \tan^2\theta &\checkmark \end{aligned}$$

8. $\frac{\tan^2\theta}{\tan^2\theta + 1} = \sin^2\theta$

$$\frac{\tan^2\theta}{\sec^2\theta} = \frac{\sin^2\theta}{\cos^2\theta} \cdot \frac{\cos^2\theta}{1} = \sin^2\theta$$

4. $\cos^2\beta - \sin^2\beta = 1 - 2\sin^2\beta$

$$\begin{aligned} 1 - \sin^2\beta - \sin^2\beta &= \\ 1 - 2\sin^2\beta &\checkmark \end{aligned}$$

9. $(\sin\alpha + \cos\alpha)^2 + (\sin\alpha - \cos\alpha)^2 = 2$

$$\begin{aligned} \sin^2\alpha + 2\sin\alpha\cos\alpha + \cos^2\alpha + \sin^2\alpha - 2\sin\alpha\cos\alpha + \cos^2\alpha &= \\ 2(\sin^2\alpha + \cos^2\alpha) &= 2(1) = 2 \end{aligned}$$

5. $\frac{\csc^2\theta}{\cot\theta} = \csc\theta \sec\theta$

$$\begin{aligned} \csc^2\theta \cdot \tan\theta &= \\ \frac{1}{\sin^2\theta} \cdot \frac{\sin\theta}{\cos\theta} &= \\ \frac{1}{\sin\theta} \cdot \frac{1}{\cos\theta} &= \\ \csc\theta \sec\theta &\checkmark \end{aligned}$$

10. $\cos^3\theta + \sin^2\theta \cos\theta = \cos\theta$

$$\begin{aligned} \cos\theta (\cos^2\theta + \sin^2\theta) &= \\ \cos\theta (1) &= \cos\theta \end{aligned}$$