

Day #47 Homework

For problems 1 – 12, find the indefinite integrals below.

$$1. \int (\sqrt[3]{x} + 3) dx = \int (x^{1/3} + 3) dx$$

$$= \frac{x^{4/3}}{4/3} + 3x + C$$

$$= \frac{3}{4} x^{4/3} + 3x + C$$

$$2. \int (2x - 3x^2) dx$$

$$= \frac{2x^2}{2} - \frac{3x^3}{3} + C$$

$$= x^2 - x^3 + C$$

$$3. \int x^2(2x^2 + 3x) dx = \int (2x^4 + 3x^3) dx$$

$$= \frac{2x^5}{5} + \frac{3x^4}{4} + C$$

$$= \frac{2}{5} x^5 + \frac{3}{4} x^4 + C$$

$$4. \int (x^{3/2} + 2x + 1) dx$$

$$= \frac{x^{5/2}}{5/2} + \frac{2x^2}{2} + x + C$$

$$= \frac{2}{5} x^{5/2} + x^2 + x + C$$

$$5. \int \left(\sqrt{x} + \frac{1}{2\sqrt{x}} \right) dx = \int \left(x^{1/2} + \frac{1}{2} x^{-1/2} \right) dx$$

$$= \frac{x^{3/2}}{3/2} + \frac{\frac{1}{2} x^{1/2}}{\frac{1}{2}} + C$$

$$= \frac{2}{3} x^{3/2} + x^{1/2} + C$$

$$6. \int \frac{3x^2 - 2x + 3}{x^3} dx = \int (3x^{-1} - 2x^{-2} + 3x^{-3}) dx$$

$$= 3 \ln x - \frac{2x^{-1}}{-1} + \frac{3x^{-2}}{-2} + C$$

$$= 3 \ln x + 2x^{-1} - \frac{3}{2} x^{-2} + C$$

$$= \frac{3x \ln x + 2x - \frac{3}{2}}{x^2} + C$$

$$7. \int y^3 \sqrt{y} dy = \int (y^3 \cdot y^{1/2}) dy = \int y^{7/2} dy$$

$$= \frac{y^{9/2}}{9/2} + C$$

$$= \frac{2}{9} y^{9/2} + C$$

$$8. \int \frac{1}{w\sqrt{w}} dw = \int (w^{-1} \cdot w^{-1/2}) dw = \int w^{-3/2} dw$$

$$= \frac{w^{-1/2}}{-1/2} + C = \frac{-2}{\sqrt{w}} + C$$

$$9. \int \frac{x^3+3}{\sqrt{x}} dx = \int \frac{x^3+3}{x^{1/2}} dx = \int (x^{5/2} + 3x^{-1/2}) dx$$

$$= \frac{x^{7/2}}{7/2} + \frac{3x^{1/2}}{1/2} + C$$

$$= \frac{2}{7}x^{7/2} + 6x^{1/2} + C$$

$$10. \int (x+3)(x-3)^2 dx$$

$$= \int (x+3)(x^2-6x+9) dx$$

$$= \int (x^3-6x^2+9x+3x^2-18x+27) dx$$

$$= \int (x^3-3x^2-9x+27) dx$$

$$= \frac{x^4}{4} - \frac{3x^3}{3} - \frac{9x^2}{2} - 27x + C$$

$$= \frac{1}{4}x^4 - x^3 - \frac{9}{2}x^2 - 27x + C$$

$$11. \int (\theta^2 + \cos \theta) d\theta$$

$$= \frac{\theta^3}{3} + \sin \theta + C$$

$$12. \int (\sqrt{x} - \sin x + 2) dx$$

$$= \int (x^{1/2} - \sin x + 2) dx$$

$$= \frac{x^{3/2}}{3/2} - (-\cos x) + 2x + C$$

$$= \frac{2}{3}x^{3/2} + \cos x + 2x + C$$

For problems 13 and 14, find the indicated function based on the given information.

13. If $f'(x) = 2x - \sin x$ and $f(0) = 4$, find $f(x)$.

$$f(x) = \int f'(x) dx = \int (2x - \sin x) dx$$

$$= \frac{2x^2}{2} - (-\cos x) + C$$

$$= x^2 + \cos x + C$$

$$f(0) = 0^2 + \cos 0 + C = 4$$

$$1 + C = 4$$

$$C = 3$$

so:

$$f(x) = x^2 + \cos x + 3$$

14. If $f''(x) = x^2$, $f'(0) = 6$, and $f(0) = 3$, find $f(x)$.

$$f'(x) = \int f''(x) dx = \int x^2 dx$$

$$f'(x) = \frac{x^3}{3} + C \quad f'(0) = \frac{0^3}{3} + C = 6$$

$$C = 6$$

$$f'(x) = \frac{1}{3}x^3 + 6$$

$$f(x) = \int (\frac{1}{3}x^3 + 6) dx$$

$$= \frac{1}{3} \cdot \frac{x^4}{4} + 6x + C = \frac{1}{12}x^4 + 6x + C$$

$$f(0) = \frac{1}{12}(0)^4 + 6(0) + C = 3 \quad \text{so } C = 3$$

$$f(x) = \frac{1}{12}x^4 + 6x + 3$$