

Consider the function below to find each limit. If a limit does not exist, state why.

$$G(x) = \begin{cases} 2x^2 + 3x, & x < -2 \\ -\frac{1}{2}x + 1, & x > -2 \end{cases}$$

a) $\lim_{x \rightarrow -2^-} G(x)$

b) $\lim_{x \rightarrow -2^+} G(x)$

c) $\lim_{x \rightarrow -2} G(x)$

Find each of the following limits analytically. Show your algebraic analysis.

a. $\lim_{x \rightarrow e} \frac{\ln x}{2x} = \frac{\ln e}{2e} = \frac{1}{2e}$

b. $\lim_{x \rightarrow 5^-} \left(\frac{2}{5}x^2 + 2x \right) = \frac{2}{5}(5)^2 + 2(5) = \frac{2}{5}(25) + 10 = 10 + 10 = 20$

c. $\lim_{\theta \rightarrow \pi} (\sin^2 \theta + 2 \cos \theta) = (\sin \pi)^2 + 2 \cos \pi = 0^2 + 2(-1) = -2$

d. $\lim_{\alpha \rightarrow \frac{5\pi}{3}} \frac{\tan \alpha}{\alpha^2} = \frac{\tan \frac{5\pi}{3}}{\left(\frac{5\pi}{3}\right)^2} = \frac{-\sqrt{3}}{\frac{25\pi^2}{9}} = -\frac{9\sqrt{3}}{25\pi^2}$

e. $\lim_{x \rightarrow -2} \frac{x^2 - x - 6}{2x + 4} = \lim_{x \rightarrow -2} \frac{(x-3)(x+2)}{2(x+2)} = \lim_{x \rightarrow -2} \frac{x-3}{2} = \frac{-2-3}{2} = -\frac{5}{2}$



$$f. \lim_{x \rightarrow 3} \frac{x+5}{x^2-9} = \lim_{x \rightarrow 3} \frac{x+5}{(x+3)(x-3)} \quad \lim_{x \rightarrow 3} \frac{x+5}{(x+3)(x-3)}$$

↪ D.N.E

$$a^3 - b^3 = (a-b)(a^2 + ab + b^2)$$

$$g. \lim_{x \rightarrow \frac{3}{2}} \frac{8x^3 - 27}{2x - 3} = \lim_{x \rightarrow \frac{3}{2}} \frac{(2x-3)(4x^2 - 6x + 9)}{2x-3} = \lim_{x \rightarrow \frac{3}{2}} 4x^2 - 6x + 9$$

$$a = 2x \quad b = 3$$

$$= 4\left(\frac{3}{2}\right)^2 - 6\left(\frac{3}{2}\right) + 9 = 4\left(\frac{9}{4}\right) + 9 + 9 = 9 + 9 + 9 = 27$$

$$h. \lim_{x \rightarrow -2} \frac{\sqrt{2x+5}-1}{x+2} \cdot \frac{\sqrt{2x+5}+1}{\sqrt{2x+5}+1} = \lim_{x \rightarrow -2} \frac{2x+5-1}{(x+2)(\sqrt{2x+5}+1)}$$

$$= \lim_{x \rightarrow -2} \frac{2x+4}{(x+2)(\sqrt{2x+5}+1)} = \lim_{x \rightarrow -2} \frac{2(x+2)}{(x+2)(\sqrt{2x+5}+1)} = \lim_{x \rightarrow -2} \frac{2}{\sqrt{2x+5}+1}$$

$$= \frac{2}{\sqrt{2(-2)+5}+1} = \frac{2}{\sqrt{-4+5}+1} = \frac{2}{1+1} = 1$$

$$i. \lim_{x \rightarrow 1} \frac{1 - \sqrt{2x^2 - 1}}{x - 1} \cdot \frac{(1 + \sqrt{2x^2 - 1})}{1 + \sqrt{2x^2 - 1}} = \lim_{x \rightarrow 1} \frac{1 - (2x^2 - 1)}{(x-1)(1 + \sqrt{2x^2 - 1})}$$

$$= \lim_{x \rightarrow 1} \frac{1 - 2x^2 + 1}{(x-1)(1 + \sqrt{2x^2 - 1})} = \lim_{x \rightarrow 1} \frac{-2x^2 + 2}{(x-1)(1 + \sqrt{2x^2 - 1})} = \lim_{x \rightarrow 1} \frac{-2(x+1)(x-1)}{(x-1)(1 + \sqrt{2x^2 - 1})}$$

$$= \frac{-2(1+1)}{1 + \sqrt{2(1)^2 - 1}} = \frac{0}{1 + \sqrt{0}} = 0$$

$$j. \lim_{x \rightarrow 0} \frac{\frac{1}{x+2} + \frac{1}{x}}{x} = \lim_{x \rightarrow 0} \frac{\frac{x}{x(x+2)} + \frac{x+2}{x(x+2)}}{x} = \lim_{x \rightarrow 0} \frac{x+x+2}{x(x+2)} \cdot \frac{x}{1}$$

$$\lim_{x \rightarrow 0} \frac{2x+2}{x(x+2)} \cdot \frac{x}{x} = \lim_{x \rightarrow 0} \frac{2x+2}{x^2(x+2)} \rightarrow \text{DNE}$$

$$k. \lim_{x \rightarrow 2^+} \frac{3x^2 + 7x + 2}{x^2 - 4} = \lim_{x \rightarrow 2^+} \frac{(3x+1)(x+2)}{(x+2)(x-2)} = \lim_{x \rightarrow 2^+} \frac{3x+1}{x-2}$$

if $x = 2.1 \rightarrow \frac{3(2.1)+1}{2.1-2} = \frac{7.3}{0.1} \rightarrow \frac{+}{+}$

so: $\lim_{x \rightarrow 2^+} \frac{3x+1}{x-2} = \infty$

$$l. \lim_{x \rightarrow 3^+} \frac{2x+5}{x-3} \rightarrow \text{TRY } x = 3.1$$

$$\frac{2(3.1)+5}{3.1-3} = \frac{+}{+}$$

$$\text{so: } \lim_{x \rightarrow 3^+} \frac{2x+5}{x-3} = \infty$$

$$m. \lim_{x \rightarrow 3^-} \frac{2x+5}{x-3} = -\infty$$

$$x = 2.9$$

$$\frac{2(2.9)+5}{2.9-3} = \frac{+}{-}$$

If $f(x) = 2x^2 - 3x + 4$, find $\lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$.