Limits of Exponential Functions

Graphical and Analytical Connections

Consider the four exponential functions graphed below. Find the indicated limits for each function based on the graph.

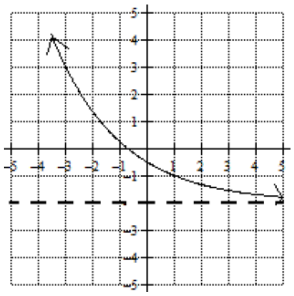
$a=1$ $b=2/3$ $f(x)=ab^x$

DECAY

H.A $y=-2$

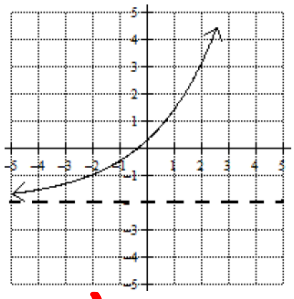
GROWTH

$f(x)=\left(\frac{2}{3}\right)^{x-1}-2$



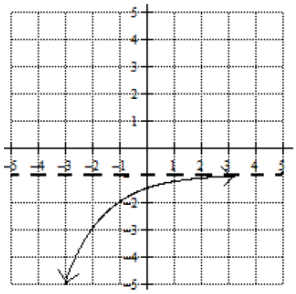
$\lim_{x\to-\infty} f(x)=\infty$ $\lim_{x\to\infty} f(x)=-2$

$f(x)=\left(\frac{3}{2}\right)^{x+2}-2$



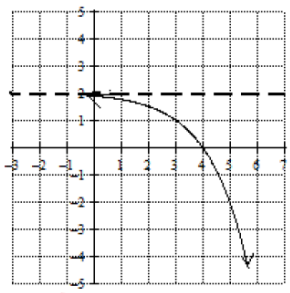
$\lim_{x\to-\infty} f(x)=-2$ $\lim_{x\to\infty} f(x)=\infty$

$f(x)=-\left(\frac{1}{2}\right)^{x+1}-1$



$\lim_{x\to-\infty} f(x)=-\infty$ $\lim_{x\to\infty} f(x)=-1$

$f(x)=-\left(\frac{1}{2}\right)^{-x+3}+2$



$\lim_{x\to-\infty} f(x)=2$ $\lim_{x\to\infty} f(x)=-\infty$

In order to determine a limit as x approaches $-\infty$ or ∞ for an exponential function, you have to determine what the graph will look like. Based on what we have seen above, what are the three possible results of such a limit for an exponential function?

$-\infty$ ∞ VALUE OF HORIZONTAL ASYMPTOTE

By studying the graphs above, remind yourself of the four rules determining if the function will be a growth or decay function. $f(x)=ab^x$

1. If a and x are both positive or both negative and $b>1$; Growth
2. If a and x are both positive or both negative and $0<b<1$; Decay
3. If a is positive and x is negative (or vice-versa) and $b>1$; Growth
4. If a is positive and x is negative (or vice-versa) and $0>b>1$; Decay

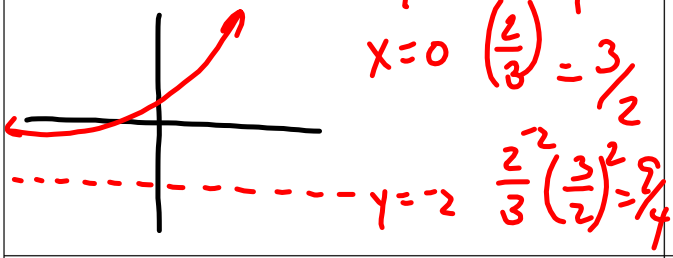
Determine the limits of each of the following exponential functions.

1. $\lim_{x\to\infty}\left(\frac{2}{3}\right)^{-x-1}-2 = \infty$

H.A $y=-2$

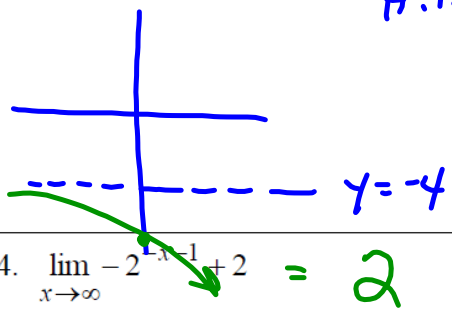
$x=0$ $\left(\frac{2}{3}\right)^{-1}=\frac{3}{2}$

$\frac{2}{3}^{-2}\left(\frac{3}{2}\right)^{2}=\frac{9}{4}$

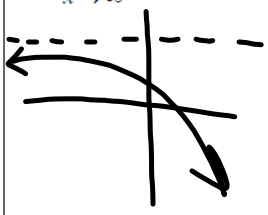


2. $\lim_{x\to-\infty}-(0.4)^{x}-4 = -4$

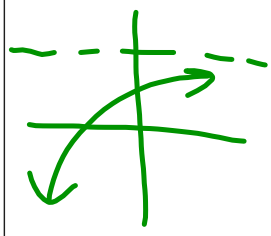
H.A $y=-4$



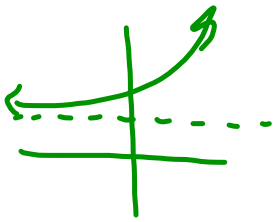
3. $\lim_{x\to\infty}-\left(\frac{2}{3}\right)^{-x+2}+3 = -\infty$



4. $\lim_{x\to\infty}-2^{-x-1}+2 = 2$



5. $\lim_{x\to-\infty}e^{-x-1}+2 = 2$



6. $\lim_{x\to\infty}-(0.4)^{x}-4 = -\infty$

7. $\lim_{x\to3}\left(e^{2-x}+2\right) = e^{2-3}+2$

$e^{-1}+2$

$\frac{1}{e}+2=\frac{1+2e}{e}$

8. $\lim_{x\to-2}\left[\left(\frac{1}{2}\right)^{-x-3}+3\right]$

$\frac{1}{2}^{2-3}+3=\frac{1}{2}^{-1}+3=2+3$

$=5$