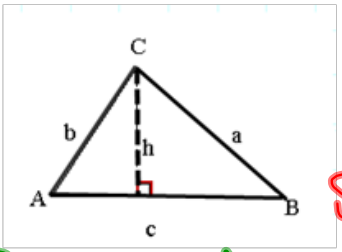
The Law of Sines

Students will utilize the Law of Sines to find the missing sides and angles of acute and obtuse triangles.

What is the Law of Sines?

Law of Sines

Used to find the missing sides and angles of oblique (non-right) triangles. Right triangle trigonometry won't work.



$\sin B = \frac{h}{a}$

$h = a \cdot \sin B$

$\sin A = \frac{h}{b}$

$h = b \sin A$

$a \sin B = b \sin A$

$\frac{a}{\sin A} = \frac{b}{\sin B}$

In any Δ ABC with angles A, B, and C and opposite sides a, b, and c the following equation is true:

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c} \quad \text{or} \quad \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

The Law of Sines can be used to solve the following cases of oblique triangles:

1. **Angle-Angle-Side: AAS**
2. **Angle-Side-Angle: ASA (side is included)**
3. **Side-Side-Angle: SSA (special case)**

Law of sines can be used for both acute and obtuse triangles.

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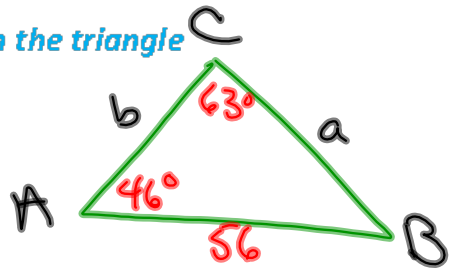
Example 1: AAS

Solve the ΔABC if m∠A = 46°, m∠C = 63°, c = 56

a. Sketch the triangle



b. Find the measure of angle B.

$B = 180° - 46° - 63° = 71°$

c. Solve for side a and side b.

SIDE a

$$\frac{a}{\sin A} = \frac{c}{\sin C}$$

$$\frac{1}{\sin 46°}\left(\frac{a}{\sin 46°}\right) = \left(\frac{56}{\sin 63°}\right)\frac{\sin 46°}{1}$$

$$a = \frac{56 \sin 46°}{\sin 63°}$$

$a \approx 45.21$

SIDE b

$$\frac{b}{\sin B} = \frac{c}{\sin C}$$

$$\frac{b}{\sin 71°} = \frac{56}{\sin 63°}$$

$$b = \frac{56 \sin 71°}{\sin 63°}$$

$b \approx 59.43$

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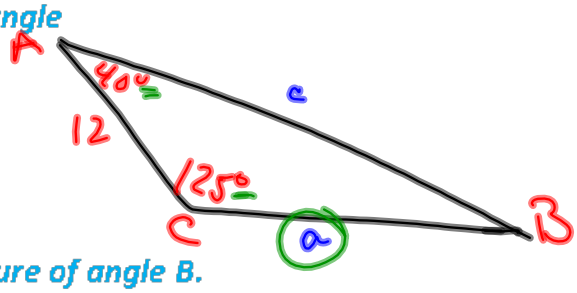
Example 2: ASA (side included)

Solve ΔABC if m∠A = 40˚, m∠C = 125˚, b. = 12

a. Sketch the triangle

$$\frac{c}{\sin C} = \frac{a}{\sin A}$$

Given



b. Find the measure of angle B.

$B = 180° - 125° - 40° = 15°$

c. Solve for side a and side c.

SIDE c

$$\frac{c}{\sin 125°} = \frac{12}{\sin 15°}$$

$$c = \frac{12 \sin 125°}{\sin 15°}$$

$c \approx 37.98$

SIDE a

$$\frac{a}{\sin 40°} = \frac{12}{\sin 15°}$$

$$a = \frac{12 \sin 40°}{\sin 15°}$$

$a \approx 29.8$

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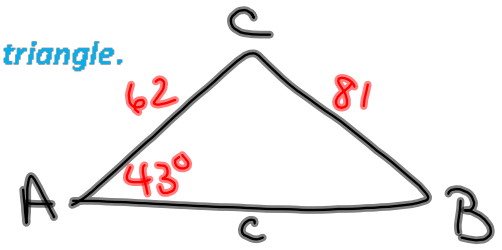
Example 3: SSA ("the ambiguous case")

Solve ΔABC if m∠A = 43°, a = 81, b = 62

a. Sketch the triangle.



b. Find the measure of angle B and determine the number of solutions.

$$\frac{\sin B}{\cancel{62}_{1}} = \frac{\sin 43^\circ}{81} \qquad 62$$

$$\sin B = \frac{62 \sin 43^\circ}{81}$$

$$B = \sin^{-1}\left(\frac{62 \sin 43^\circ}{81}\right) \approx \boxed{31.5^\circ}$$

$$C = 180^\circ - 31.5 - 43$$

$$\boxed{C \approx 105.5^\circ}$$

2Δs?

① FIND SUPP TO 31.5°

180 − 31.5

= 148.5°

② + 43°

191.5°

> 180 (1Δ)

< 180 (2Δ)

c. Solve for side c.

$$\frac{c}{\cancel{\sin 105.5}} = \frac{81 \sin 105.5}{\sin 43^\circ}$$

$$c \approx 114.4$$

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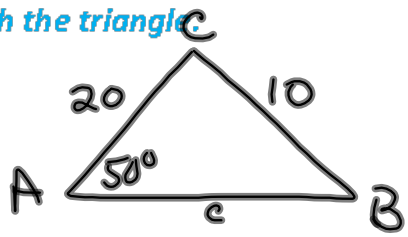
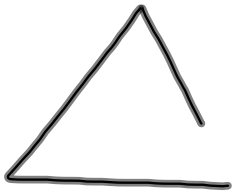
Example 4: SSA ("the ambiguous case")

Solve ΔABC if m∠A = 50°, a = 10, b = 20

a. Sketch the triangle





b. Find the measure of angle B and determine the number of solutions.

$$\frac{\sin B}{\cancel{20}} = \frac{20 \sin 50^\circ}{10}$$

$$B = \sin^{-1}\left(\frac{20 \sin 50^\circ}{10}\right) = \text{UNDEF}$$

NO Δ EXISTS

c. Solve for side c.

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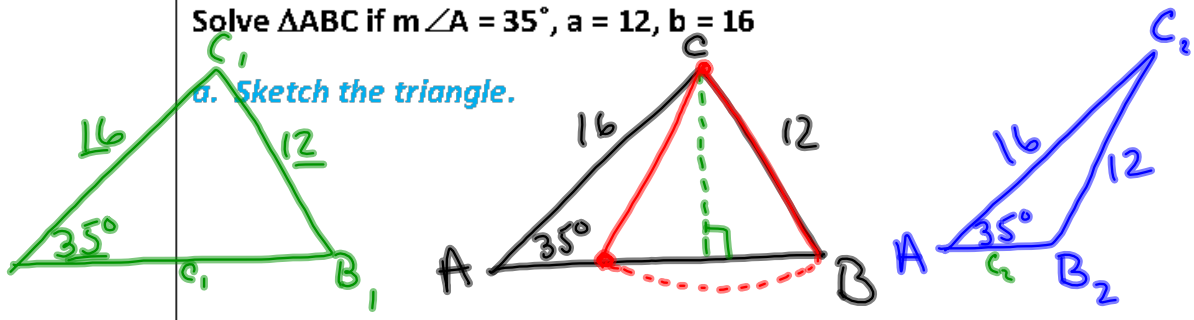
Example 5: SSA ("the ambiguous case")

Solve ΔABC if m∠A = 35°, a = 12, b = 16

a. Sketch the triangle.



b. Find the measure of angle B and determine the number of solutions.

2 Δs ?

180−50

130°

+ 35

165°

<180 2Δ

$$\frac{\sin B}{\cancel{16}} \overset{?}{=} \frac{16\sin 35°}{12}$$

$$B_1 = \sin^{-1}\left(\frac{16\sin 35°}{12}\right) \approx 50°$$

$$C_1 = 180 - 50 - 35 = 95°$$

$$B_2 = 130°$$

$$C_2 = 180 - 130 - 35 = 15°$$

c. Solve for side c.

$$\frac{c_1}{\sin 95°} = \frac{12}{\sin 35°}$$

$$c_1 = \frac{12\sin 95°}{\sin 35°}$$

$$c_1 \approx 20.8$$

$$\frac{c_2}{\sin 15°} = \frac{12}{\sin 35°}$$

$$c_2 = \frac{12\sin 15°}{\sin 35°}$$

$$c_2 \approx 5.4$$

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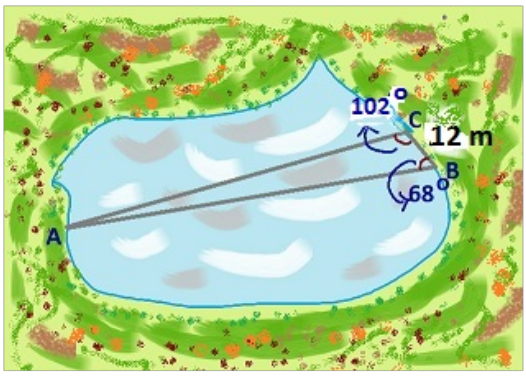
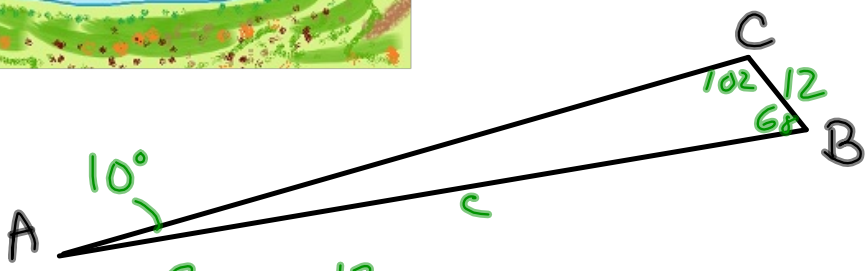
Example 6: Applications

In a children's amusement park, the administration wanted to build a walker's bridge along a Pond in the midst of the park from point A to B. For this purpose an Engineer identified the approximate straight edge of the Pond near B and marked a point C at a distance 12 m from B. He was able to measure the angles ACB and ABC as 102° and 68° to estimate the length of the bride to be built. Find the approximate length of the bridge rounded to a meter.





$$\frac{c}{\sin 102} = \frac{12}{\sin 10^\circ}$$

$$c = \frac{12 \sin 102^\circ}{\sin 10^\circ} \approx 68\text{m}$$

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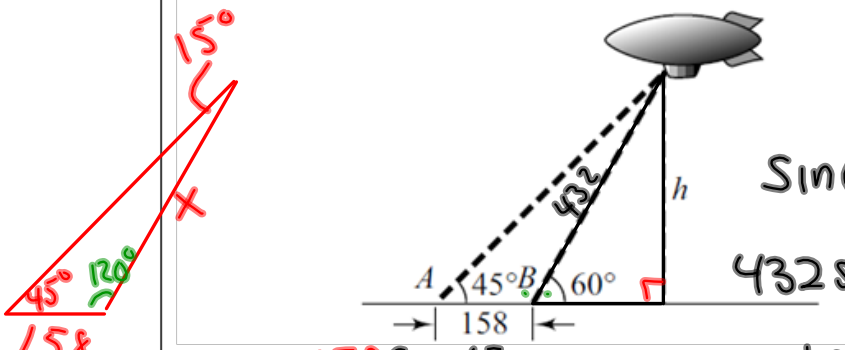
Example 7: Applications

Height of Blimp In order to find the height of the Goodyear blimp, observers at A and B, 158 yards apart, measure the following angles: $\alpha = 45.0°$ and $\beta = 60.0°$. (See the diagram.) How high is the blimp?



$\sin 60° = \frac{h}{432}$

$432 \sin 60° = h$

$h \approx 374$ yd

$158 \sin 45$

$\frac{x}{\sin 45°} = \frac{158}{\sin 15°}$

$x = \frac{158 \sin 45°}{\sin 15°} \approx 432$ yd

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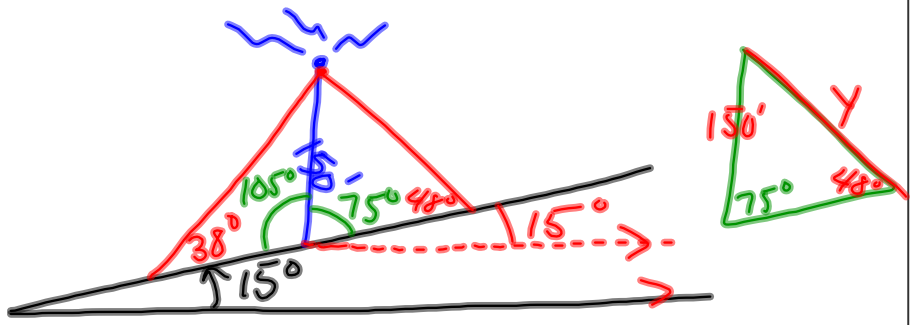
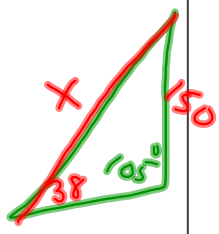
Example 8: Applications

An antenna is to be placed on a hillside that has a slope of 15°. Guywires are to be placed 150 feet up on the tower. If the angle of elevation of the downhill wire is 38° and the angle of elevation of the uphill wire is 48°, how long is each wire?





$$\frac{x}{\sin 105} = \frac{150}{\sin 38}$$

$x \approx 235.3'$

$$\frac{y}{\sin 75^\circ} = \frac{150}{\sin 48^\circ}$$

$y \approx 195'$

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