

Law of Sines Review Problems

Use the Law of Sines to solve the following triangles. If two triangles exist, solve both of them.

1. $\triangle ABC$; $A = 40^\circ$, $B = 30^\circ$, $b = 10$

$$\begin{aligned} C &= 110^\circ \\ a &\approx 12.86 \\ c &\approx 18.79 \end{aligned}$$

2. $\triangle BCD$; $B = 16^\circ$, $C = 103^\circ$, $c = 12$

$$\frac{b}{\sin 16^\circ} = \frac{12 \sin 16^\circ}{\sin 103^\circ}$$

$$b \approx 3.39$$

$$\frac{a}{\sin 61^\circ} = \frac{12 \sin 61^\circ}{\sin 103^\circ}$$

$$a \approx 10.77$$

$$\begin{aligned} A &= 61^\circ \\ b &\approx 3.39 \\ a &\approx 10.77 \end{aligned}$$

$$\frac{10 \sin 40^\circ}{\sin 30^\circ} = \frac{a}{\sin 110^\circ}$$

$$a \approx 12.86$$

$$\frac{c}{\sin 110^\circ} = \frac{10 \sin 110^\circ}{\sin 30^\circ} \quad c \approx 18.79$$

3. $\triangle ABC$; $A = 32^\circ$, $a = 17$, $b = 11$

ONE Δ

$$\begin{aligned} B &= 20^\circ \\ C &= 128^\circ \\ c &\approx 25.28 \end{aligned}$$

$$\frac{11 \sin 32^\circ}{\sin B} = \frac{17}{\sin 32^\circ}$$

$$B = \sin^{-1}\left(\frac{11 \sin 32^\circ}{17}\right) \approx 20^\circ$$

$$\frac{c}{\sin 128^\circ} = \frac{17 \sin 128^\circ}{\sin 32^\circ}$$

4. $\triangle EFG$; $F = 73^\circ$, $f = 24$, $g = 28$

$$\frac{28 \sin 73^\circ}{24} = \frac{g \sin 73^\circ}{24}$$

$$G = \sin^{-1}\left(\frac{28 \sin 73^\circ}{24}\right)$$

$$G \approx \text{undefined}$$

NO Δ

5. $\triangle PQR$; $P = 30^\circ$, $q = 18$, $p = 9$

$$\frac{\sin Q}{18} = \frac{\sin 30^\circ}{9}$$

$$Q = 90^\circ$$

$$18^2 - 9^2 = r^2$$

$$r \approx 15.59$$

$$\begin{aligned} Q &= 90^\circ \\ R &= 60^\circ \\ r &= 15.59 \end{aligned}$$

$$107.26$$

$$\frac{\sin K}{17} = \frac{\sin 64^\circ}{16}$$

$$K \approx 72.74^\circ$$

$$\frac{l_1}{\sin 73.26^\circ} = \frac{16 \sin 43.26^\circ}{\sin 64^\circ}$$

$$\frac{l_2}{\sin 107.26^\circ} = \frac{16 \sin 107.26^\circ}{\sin 64^\circ}$$

6. $\triangle KLI$; $J = 64^\circ$, $j = 16$, $k = 17$

2 Δ s

$$K_1 \approx 72.74^\circ$$

$$K_2 \approx 107.26^\circ$$

$$L_1 \approx 43.26^\circ$$

$$L_2 \approx 8.74^\circ$$

$$l_1 \approx 12.2$$

$$l_2 \approx 2.8$$

7. $\triangle ABC$; $A = 136^\circ$, $a = 15$, $b = 28$

$$\frac{\sin B}{28} = \frac{\sin 136^\circ}{15}$$

NO Δ

$$B \approx \text{undefined}$$

$$\frac{u}{\sin 90^\circ} = \frac{22 \sin 19^\circ}{\sin 47^\circ}$$

$$\frac{a}{\sin 114^\circ} = \frac{22 \sin 114^\circ}{\sin 47^\circ}$$

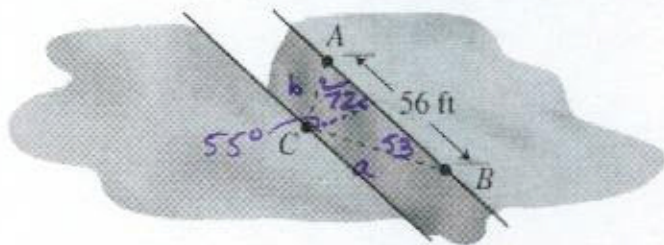
$$A = 114^\circ$$

$$u \approx 9.79$$

$$a \approx 27.48$$

8. $\triangle USA$; $U = 19^\circ$, $s = 22$, $S = 47^\circ$

9. Two markers A and B on the same side of a canyon rim are 56 feet apart. A third marker C, located across the rim, is positioned so that angle $BAC = 72^\circ$ and angle $ABC = 53^\circ$.



- a. Find the distance between C and A. (b)

$$\frac{b}{\sin 53^\circ} = \frac{56 \sin 53^\circ}{\sin 55^\circ}$$

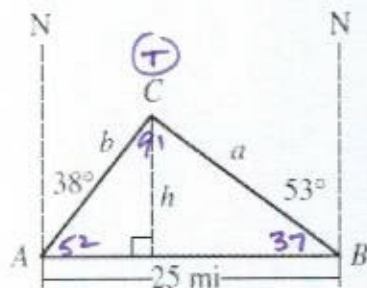
$$b \approx 54.6 \text{ ft}$$

- b. Find the distance between the two canyon rims (Assume they are parallel).

$$\sin 72^\circ = \frac{d}{b}$$

$$d = b \sin 72^\circ \approx 51.9 \text{ ft}$$

10. Two meteorologists are 25 miles apart, located on an East-West road. The meteorologist at point A sights a tornado 38° East of North. The meteorologist at point B sights the same tornado at 53° West of North. Find the distance from each meteorologist to the tornado. Also find the distance between the tornado and the road.



$$\frac{a}{\sin 52^\circ} = \frac{25 \sin 52^\circ}{\sin 91^\circ}$$

$$a \approx 19.7 \text{ mi}$$

$$\frac{b}{\sin 37^\circ} = \frac{25 \sin 37^\circ}{\sin 91^\circ}$$

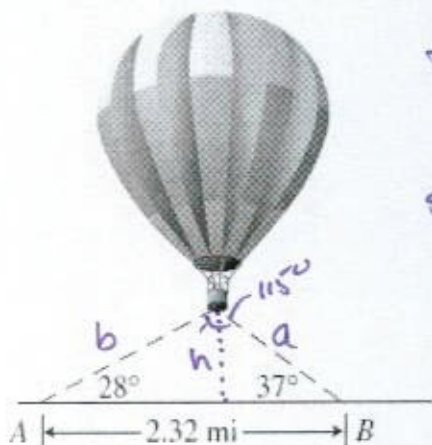
$$b \approx 15.1 \text{ mi}$$

$$\sin 37^\circ = \frac{h}{a}$$

$$h = a \sin 37^\circ \quad \text{or} \quad h = b \sin 52^\circ$$

$$h \approx 11.9 \text{ mi}$$

11. Observers 2.32 miles apart see a hot-air balloon directly between them, but at the angles of elevation shown in the figure. Find the altitude of the balloon.



find either a or b ...

$$\frac{a}{\sin 28^\circ} = \frac{2.32}{\sin 115^\circ}$$

$$a \approx 1.2 \text{ mi}$$

$$\frac{b}{\sin 37^\circ} = \frac{2.32}{\sin 115^\circ}$$

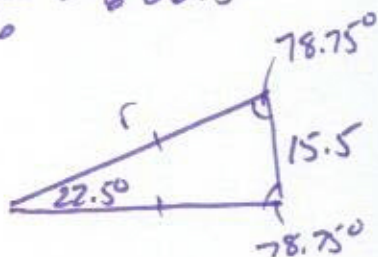
$$b \approx 1.5 \text{ mi}$$

$$h = a \sin 37^\circ \quad \text{or} \quad h = b \sin 28^\circ$$

$$h \approx 0.72 \text{ mi} (\approx 3819 \text{ ft})$$

12. A Ferris wheel has 16 evenly spaced cars. The distance between adjacent chairs is 15.5 feet. Find the radius of the wheel (to the nearest 0.1 feet).

$$\frac{360}{16} = 22.5^\circ$$



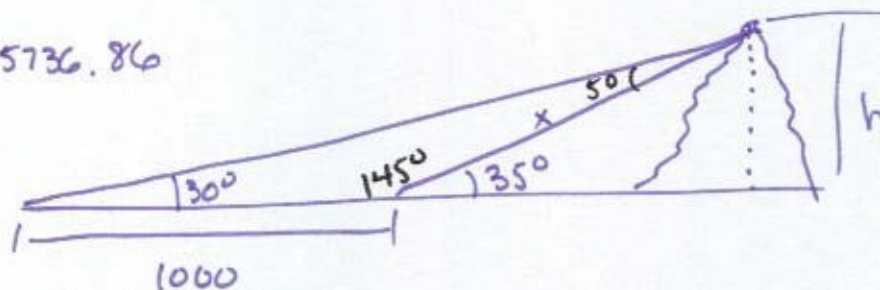
$$\frac{r}{\sin 78.75^\circ} = \frac{15.5}{\sin 22.5^\circ}$$

$$r \approx 39.7 \text{ ft}$$

13. While hiking on a level path toward Colorado's front range, Otis Evans determines that the angle of elevation to the top of Long's Peak is 30° . Moving 1000 feet closer to the mountain, Otis determines the angle of elevation to be 35° . How much higher is the top of Long's Peak than Otis's elevation?

$$\frac{x}{\sin 30^\circ} = \frac{1000}{\sin 5^\circ}$$

$$x = \frac{500}{\sin 5^\circ} \approx 5736.86$$

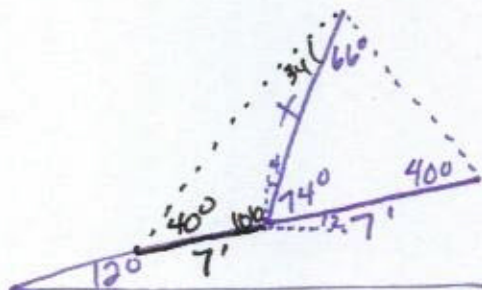


$$\sin 35^\circ = \frac{h}{x}$$

$$h = x \sin 35^\circ$$

$$h \approx 3290.53 \text{ ft}$$

14. A street sign that is on a hill with a 12° incline is leaning at a 4° angle from the vertical. It casts a shadow that is 7 feet long when the sun's angle of elevation is 40° . What is the length of the street sign's post? (How would your answer change if the sun is on the other side of the sign?)



$$\frac{x}{\sin 40^\circ} = \frac{7}{\sin 66^\circ}$$

$$x = \frac{7 \sin 40^\circ}{\sin 66^\circ} \approx 5.2 \text{ ft}$$

$$\frac{x}{\sin 40^\circ} = \frac{7}{\sin 34^\circ}$$

$$x = \frac{7 \sin 40^\circ}{\sin 34^\circ} \approx 8.1 \text{ ft}$$