1. $\lim_{x \to 0} \frac{2x^6 + 6x^3}{4x^5 + 3x^3}$ is

(A) 0 (B) $\frac{1}{2}$ (C) 1 (D) 2 (E) nonexistent

$$\lim_{x \to 0} \frac{2\cancel{x^3}(x^3+3)}{\cancel{x^3}(4x^2+3)} = \lim_{x \to 0} \frac{2(x^3+3)}{4x^2+3} = \frac{2[(0)^3+3]}{4(0)^2+3} = \frac{6}{3} = 2$$

$$f(x) = \begin{cases} x^2 - 3x + 9 & \text{for } x \le 2 \\ kx + 1 & \text{for } x > 2 \end{cases}$$

2. The function f is defined above. For what value of k, if any, is f continuous at $x = 2$?

(A) 1

(B) 2

(C) 3

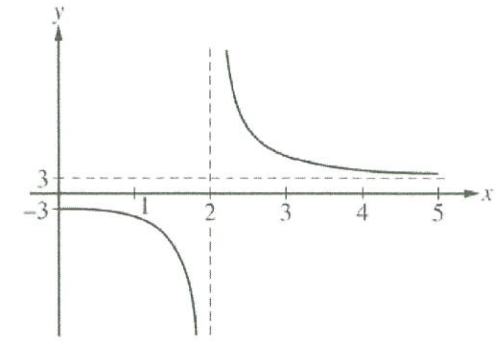
(D) 7

(E) No value of k will make f continuous at $x = 2$.

$$\lim_{x \to 2^-} f(x) = \lim_{x \to 2^+} f(x)$$

$x=2$

$$x^2 - 3x + 9 = kx + 1$$
$$2^2 - 3(2) + 9 = 2k + 1$$
$$4 - 6 + 9 = 2k + 1$$
$$7 = 2k + 1$$
$$6 = 2k$$
$$3 = k$$



3. The function f is given by $f(x) = \dfrac{ax^2 + 12}{x^2 + b}$. The figure above shows a portion of the graph of f. Which of the following could be the values of the constants a and b?

(A) $a = -3,\ b = 2$

(B) $a = 2,\ b = -3$

(C) $a = 2,\ b = -2$

(D) $a = 3,\ b = -4$ ✓

(E) $a = 3,\ b = 4$

V.A @ $x = 2$

so $b = -4$

$\cancel{x^2 + b} \neq 0$

$\cancel{x^2} = b$

$\cancel{x} = \pm b$

---

4. If $f(x) = 3x^2 + 2x$, then $\lim\limits_{h\to 0} \dfrac{f(x+h) - f(x)}{h}$ is…

(A) $6x + 2$
(B) $6x$
(C) 0
(D) nonexistent
(E) 2

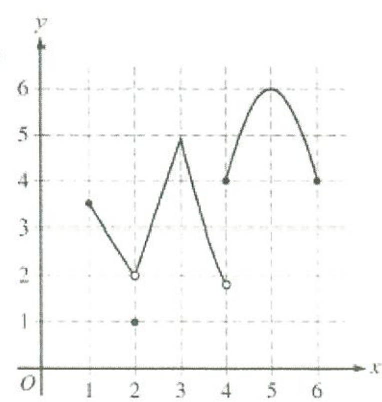
$\lim\limits_{h\to 0} \dfrac{3(x+h)^2 + 2(x+h) - (3x^2 + 2x)}{h}$

$= \lim\limits_{h\to 0} \dfrac{3(x^2 + 2xh + h^2) + 2x + 2h - 3x^2 - 2x}{h}$

$= \lim\limits_{h\to 0} \dfrac{\cancel{3x^2} + 6xh + 3h^2 + \cancel{2x} + 2h - \cancel{3x^2} - \cancel{2x}}{h}$

$\lim\limits_{h\to 0} \dfrac{\cancel{h}(6x + 3h + 2)}{\cancel{h}} \to 6(x) + 3 \cdot 0 + 2 \to 6x + 2$

5.



Graph of f

The graph of the function f is shown above. Which of the following statements is false?

(A) $\lim\limits_{x\to 2} f(x)$ exists. T

(B) $\lim\limits_{x\to 3} f(x)$ exists. T

(C) $\lim\limits_{x\to 4} f(x)$ exists. F

(D) $\lim\limits_{x\to 5} f(x)$ exists. T

(E) The function f is continuous at $x = 3$. T

$$g(x)=\begin{cases}\sin\frac{x\pi}{4}, & x<3 \\ x\sqrt{2}, & x=3 \\ \frac{x\sqrt{2}}{4x-6}, & x>3\end{cases}$$

$\sin\frac{3\pi}{4} = \frac{\sqrt{2}}{2}$

$3\sqrt{2}$

$\frac{3\sqrt{2}}{4(3)-6} = \frac{3\sqrt{2}}{12-6} = \frac{3\sqrt{2}}{6} = \frac{\sqrt{2}}{3}$

6. For the function above, which of the following would be the reason(s) why the function, $g(x)$, is not continuous at $x = 3$?

I. $g(3)$ is undefined. X

II. $\lim\limits_{x\to 3} g(x)$ does not exist. ✓

III. $\lim\limits_{x\to 3} g(x) \neq g(3)$. ✓

(A) III only
(B) II only
(C) I and II only
(D) I only
(E) II and III only

---

7. $\lim\limits_{x\to 2^+} \frac{3-2x}{x-2}$ is…

$x = 2.1$ $\quad \frac{3-2(2.1)}{2.1-2} = \frac{-}{+}$

(A) 0
(B) ∞
(C) $-\infty$
(D) 2
(E) ½

---

8. Let f be a function that is continuous on the closed interval $[2, 4]$ with $f(2) = 10$ and $f(4) = 20$. Which of the following is guaranteed by the Intermediate Value Theorem?

(A) $f(x) = 13$ has at least one solution in the open interval $(2, 4)$. $\quad f(2) < f(x) < f(4)$

$10 < f(x) < 20$

(B) $f(3) = 15$

(C) f attains a maximum on the open interval $(2, 4)$.

(D) $f(x) = 9$ has at least one solution in the open interval $(2, 4)$.

(E) $f(x)$ has at least one zero in the open interval $(2, 4)$.

AP Calculus Free Response Practice #1
Calculator Permitted

Consider the function $h(x) = \dfrac{-2x - \sin x}{x-1}$ to answer the following questions.

a. Find $\lim_{x \to \frac{\pi}{2}} [h(x) \cdot (2x-2)]$. Show your analysis.

$$\lim_{x \to \frac{\pi}{2}} h(x) \cdot \left[\lim_{x \to \frac{\pi}{2}} 2x - \lim_{x \to \frac{\pi}{2}} 2\right]$$

$$\lim_{x \to \frac{\pi}{2}} \frac{-2x - \sin x}{x-1} \left[\lim_{x \to \frac{\pi}{2}} 2x - \lim_{x \to \frac{\pi}{2}} 2\right]$$

$$\frac{-2\left(\frac{\pi}{2}\right) - \sin\frac{\pi}{2}}{\frac{\pi}{2} - 1} \left[\cancel{2}\left(\frac{\pi}{\cancel{2}}\right) - 2\right]$$

$$\frac{-\pi - 1}{\frac{\pi}{2} - 1}\left[\pi - 2\right] = \frac{-\pi - 1}{\frac{\pi - 2}{2}} \cdot \pi - 2 = -\pi - 1 \cdot \cancel{\pi - 2} \cdot \frac{2}{\cancel{\pi - 2}}$$

$$= -2\pi - 2$$

b. Identify the vertical asymptote(s), if any exist, of $h(x)$ and justify the existence by writing a limit.

V.A. @ $x = 1$

$$\lim_{x \to 1^+} \frac{-2x - \sin x}{x-1} \quad x = 1.1 \quad \frac{-2(1.1) - \sin(1.1)}{1.1 - 1} = \frac{-}{+} = -\infty$$

$$\lim_{x \to 1^-} \frac{\cancel{-2(-1) - \sin(-1)}}{-1-1} = x = 0.9 \quad \frac{-2(0.9) - \sin(0.9)}{0.9 - 1} = \frac{-}{-} = \infty$$

h(x) has a vertical asymptote @ x = 1 because the limits approaching x = 1 are −∞ & ∞

c. Identify the horizontal asymptote(s), if any exist, of $h(x)$ and justify the existence by writing a limit.

$$\lim_{x\to\infty} \frac{\frac{-2x}{x} - \frac{\sin x}{x}^{0}}{\frac{x}{x} - \frac{1}{x}} = \frac{\frac{-2\cancel{\infty}}{\cancel{\infty}} - \frac{\sin\infty}{\infty}}{1 - \frac{1}{\infty}} = \frac{-2-0}{1-0} = -2$$

since $\lim_{x\to\infty} h(x) = -2$, there is a H.A. @ $y = -2$

d. Explain why the Intermediate Value Theorem guarantees a value of c on the interval [1.5, 2.5] such that $h(c) = -4$. Then, find c.

(1) $h(x)$ is undefined at $x = 1$, which is not on $[1.5, 2.5]$, so $h(x)$ is continuous on $[1.5, 2.5]$

(2) $\cancel{\lim_{x\to c^-} h(x) = \lim_{x\to c^+} h(x)}$

$$\lim_{x\to c} h(x) = -4 \rightarrow \frac{-2c - \sin c}{c-1} = -4$$

$$-2c - \sin c = -4c - 4$$

$$2c - \sin c + 4 = 0 \quad \underline{\text{GRAPH}}$$

$$c \approx 2.35$$