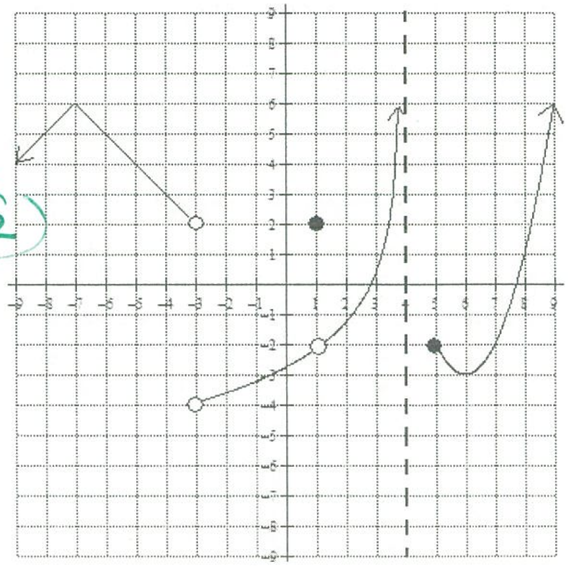
AP Calculus
Extra Practice on Limits and Multiple Choice Practice

KEY

For questions 1 – 5, refer to the graph of $f(x)$ to the right. Find the value of each indicated limit. If a limit does not exist, give a reason.



1.	$\lim\limits_{x\to-3^+} f(x) + \lim\limits_{x\to5^+} 3f(x)$	-10 $-4+3(2) =$
2.	$\lim\limits_{x\to1}\left[\frac{1}{2}f(x)+\cos(\pi x)\right]$	$\lim\limits_{x\to1}\frac{1}{2}\cdot\lim\limits_{x\to1}f(x)+\lim\limits_{x\to1}\cos\pi x$ $\frac{1}{2}(-2)+\cos\pi = -1+-1 = -2$
3.	$\lim\limits_{x\to4^-} f(x)$	DNE ∞
4.	$\lim\limits_{x\to-\infty} f(x)$	DNE $-\infty$
5.	$\lim\limits_{x\to-3} f(x)$	DNE $\lim\limits_{x\to-3^-}f(x)\neq\lim\limits_{x\to-3^+}f(x)$

For questions 6 – 11, find the value of each limit analytically. If a limit does not exist, state why.

6. $\lim\limits_{x\to0}\dfrac{x^3-2x^2+3x}{x}$

$\lim\limits_{x\to0}\dfrac{\cancel{x}(x^2-2x+3)}{\cancel{x}} = 0^2-2(0)+3$

$= 3$

7. $\lim\limits_{x\to0}\dfrac{3\tan x}{x\sec x}$ $\to 3\lim\limits_{x\to0}\dfrac{\frac{\sin x}{\cos x}}{x\frac{1}{\cos x}}$

$3\lim\limits_{x\to0}\dfrac{\sin x}{x}$ ~~$= 3(\sin 0)$~~ ~~$= 3(0) = 0$~~

$= 3(1) = 3$

8. $\lim\limits_{x\to3^+}\dfrac{x^2-4}{x^2-x-6}$ DNE

$\lim\limits_{x\to3^+}\dfrac{\cancel{(x+2)}(x-2)}{(x-3)\cancel{(x+2)}} = \dfrac{3-2}{3-3}$

$x = 3.1 \to \dfrac{3.1-2}{3.1-3} = \dfrac{+}{+} \to \infty$

9. $\lim\limits_{x\to2^-}\ln(-x+2)$ DNE

$\lim\limits_{x\to2^-}\ln(-x+2) = \infty$

10. $\lim\limits_{x\to\infty}\dfrac{x^3-2x^2+3x}{x^2-3x^3} = -\dfrac{1}{3}$

(H.A. @ $y = -\frac{1}{3}$)

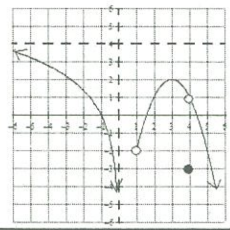
11. $\lim\limits_{x\to\infty} 5+\dfrac{5}{x} = \lim\limits_{x\to\infty}5 + \lim\limits_{x\to\infty}\dfrac{5}{x}$

$= 5 + \dfrac{5}{\infty} = 5^-$

For question 12 – 16, use the equation $g(x)$ below and the graph of the function $f(x)$.

$$g(x)=\begin{cases}3|x+3|, & x<-2\\ \cos\left(\frac{\pi x}{2}\right), & -2\le x<2\\ ax^2+2x, & x\ge 2\end{cases}$$

Graph of $f(x)$



12. Is $g(x)$ continuous at $x = -2$. [Base your response on the three part definition of continuity.]

① $g(-2) = \cos\frac{-2\pi}{2} = \cos(-\pi) = -1$

② $\lim\limits_{x\to -2^-} g(x) = 3|-2+3| = 3|1| = 3$

$\lim\limits_{x\to -2^+} g(x) = \cos\frac{-2\pi}{2} = -1$

$\lim\limits_{x\to 2^-} g(x) \neq \lim\limits_{x\to 2^+} g(x)$ SO $g(x)$ IS NOT CONTINUOUS @ $x = -2$

13. For what value(s) of a is $g(x)$ continuous at $x = 2$?

$\lim\limits_{x\to 2^-} g(x) = \lim\limits_{x\to 2^+} g(x) \Rightarrow$ ~~[illegible]~~ $\cos\left(\frac{2\pi}{2}\right) = 4a+4$

$-1 = 4a+4$

$-5 = 4a$

$\boxed{a = -\frac{5}{4}}$

14. For what value(s) of b is the function $f(x)$ discontinuous? At which of these values does $\lim\limits_{x\to b} f(x)$ exist? Explain your reasoning.

$f(x)$ IS DISCONTINUOUS AT $x = 0$: $f(0)$ IS UNDEFINED

$f(x)$ IS DISCONTINUOUS AT $x = 1$: $f(1)$ " "

$f(x)$ IS DISCONTINUOUS AT $x = 4$: $f(4) \neq \lim\limits_{x\to 4} f(x)$

15. Find $\lim\limits_{x\to 2^+}[g(x)+2f(x)]$.

$\lim\limits_{x\to 2^+} g(x) + \left(\lim\limits_{x\to 2^+} 2\right)\left(\lim\limits_{x\to 2^+} f(x)\right)$

$= a(2)^2 + 2(2) + 2(1) = 4a^2+4+2 = \boxed{4a^2+6}$

16. Which of the following limits do(es) not exist? Give a reason for your answers.

$\lim\limits_{x\to 1} f(x)$	$\lim\limits_{x\to 4} f(x)$	$\lim\limits_{x\to 0^-} f(x)$
DNE: $\lim\limits_{x\to 1^-} f(x) \neq \lim\limits_{x\to 1^+} f(x)$	$\lim\limits_{x\to 4} f(x) = 1$ EXISTS :)	DNE: $\lim\limits_{x\to 0^-} f(x) = -\infty$

17. Find the values of k and m so that the function below is continuous on the interval $(-\infty,\infty)$.

$$f(x)=\begin{cases} x^2-kx+3, & x<-2 \\ 2x-3, & -2\le x\le 3 \\ 3-2m, & x>3 \end{cases}$$

$\lim_{x\to -2^-} f(x) = \lim_{x\to -2^+} f(x)$

$(-2)^2 - k(-2)+3 = 2(-2)-3$

$4+2k+3=-4-3$

$2k=-14$

$\boxed{k=-7}$

$\lim_{x\to 3^-} f(x) = \lim_{x\to 3^+} f(x)$

$2(3)-3=3-2m$

$3=3-2m$

$0=-2m$

$\boxed{0=m}$

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18. $\lim_{x\to 0}\dfrac{4x-3}{7x+1}=$ $\dfrac{4(0)-3}{7(0)+1}=\dfrac{-3}{1}$

A. ∞ B. $-\infty$ C. 0 D. $\frac{4}{7}$ E. -3

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19. $\lim_{x\to \frac{1}{3}}\dfrac{9x^2-1}{3x-1}=$ $\lim_{x\to \frac{1}{3}} \dfrac{(3x+1)\cancel{(3x-1)}}{\cancel{3x-1}} = 3\left(\frac{1}{3}\right)+1=1+1=2$

A. ∞ B. $-\infty$ C. 0 D. 2 E. 3

$a=x$ $b=2$

20. $\lim_{x\to 2}\dfrac{x^3-8}{x^2-4}=$ $\dfrac{\cancel{(x-2)}(x^2+2x+4)}{(x+2)\cancel{(x-2)}}=\dfrac{2^2+2(2)+4}{2+2}=\dfrac{12}{4}=3$

A. 4 B. 0 C. 1 D. 3 E. 2

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21. The function $G(x)=\begin{cases} x-3, & x>2 \\ -5, & x=2 \\ 3x-7, & x<2 \end{cases}$ is not continuous at $x=2$ because…

$\lim_{x\to 2^+} = 2-3=-1$

$\lim_{x\to 2^-} = 3(2)-7=-1$

-1 -5

~~A.~~ $G(2)$ is not defined ~~B.~~ $\lim_{x\to 2} G(x)$ does not exist C. $\lim_{x\to 2} G(x)\neq G(2)$

D. Only reasons B and C E. All of the above reasons.

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22. $\lim_{x\to\infty}\dfrac{-3x^2+7x^3+2}{2x^3-3x^2+5}=$ $\rightarrow$ H.A. @ $y=\frac{7}{2}$

A. ∞ B. $-\infty$ C. 1 D. $\frac{7}{2}$ E. $-\frac{3}{2}$

23. $\lim\limits_{x\to -2} \dfrac{\sqrt{2x+5}-1}{x+2}$ $\cdot \dfrac{\sqrt{2x+5}+1}{\sqrt{2x+5}+1} = \dfrac{2x+5-1}{(x+2)(\sqrt{2x+5}+1)} = \dfrac{2x+4}{(x+2)(\sqrt{2x+5}+1)} = \dfrac{2\cancel{(x+2)}}{\cancel{(x+2)}(\sqrt{2x+5}+1)} = \dfrac{2}{\sqrt{2(-2)+5}+1} = \dfrac{2}{2} = 1$

A. 1 B. 0 C. ∞ D. $-\infty$ E. Does Not Exist

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24. If $f(x) = 3x^2 - 5x$, then find $\lim\limits_{h\to 0} \dfrac{f(x+h)-f(x)}{h}$.

$\lim\limits_{h\to 0} \dfrac{3\overset{(x^2+xh+h^2)}{(x+h)^2} - 5(x+h) - (3x^2-5x)}{h}$

$= \lim\limits_{h\to 0} \dfrac{\cancel{3x^2} + 3xh + 3h^2 \cancel{-5x} - 5h \cancel{-3x^2} \cancel{+5x}}{h}$

$= \lim\limits_{h\to 0} \dfrac{\cancel{h}(3x+3h-5)}{\cancel{h}} = 3x + 3(0) - 5$

$= 3x - 5$

A. $3x-5$
B. $6x-5$
C. $6x$
D. 0
E. Does not exist

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25. $\lim\limits_{x\to -\infty} \dfrac{2-5x}{\sqrt{x^2+2}} = \lim\limits_{x\to -\infty} \dfrac{\dfrac{2}{-x} - \dfrac{5x}{-x}}{\sqrt{\dfrac{x^2}{x^2} + \dfrac{2}{x^2}}} = \lim\limits_{x\to -\infty} \dfrac{-\dfrac{2}{x} + 5}{\sqrt{1 + \dfrac{2}{x^2}}} = \dfrac{0+5}{\sqrt{1+0}} = 5$

A. 5 B. -5 C. 0 D. $-\infty$ E. ∞

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26. The function $f(x) = \dfrac{2x^2+x-3}{x^2+4x-5}$ $\left(\dfrac{(2x+3)\cancel{(x-1)}}{(x+5)\cancel{(x-1)}}\right)$ has a vertical asymptote at $x = -5$ because…

$\dfrac{2(-4.9)+3}{-4.9+5} = \dfrac{-}{+} = -\infty$

$x = -5.1$, $x \to -5^-$: $\dfrac{2(-5.1)+3}{-5.1+5} = \dfrac{-}{-} = \infty$

~~A.~~ $\lim\limits_{x\to -5^+} f(x) = \infty$

B. $\lim\limits_{x\to -5^-} f(x) = -\infty$

C. $\lim\limits_{x\to -5^-} f(x) = \infty$

D. $\lim\limits_{x\to \infty} f(x) = -5$

E. $f(x)$ does not have a vertical asymptote at $x = -5$

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27. Consider the function $H(x) = \begin{cases} 3x-5, & x < 3 \\ x^2-2x, & x \geq 3 \end{cases}$. Which of the following statements is/are true?

$3(3)-5 = 4$

$9-6 = 3$

✓ I. $\lim\limits_{x\to 3^-} H(x) = 4$.

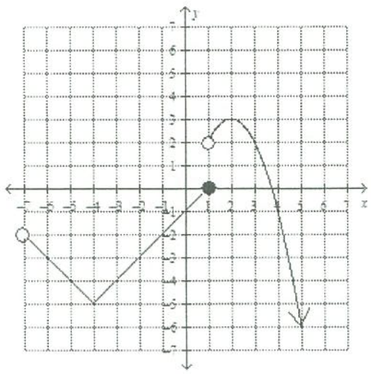
~~II.~~ $\lim\limits_{x\to 3} H(x)$ exists.

~~III.~~ $H(x)$ is continuous at $x = 3$.

A. I only B. II only C. I and II only

D. I, II and III E. None of these statements is true

AP Calculus Free Response Practice #2
Calculator NOT Permitted



Graph of $g(x)$

$$f(x) = \begin{cases} ax+3, & x<-3 \\ x^2-3x, & -3 \le x < 2 \\ bx-5, & x \ge 2 \end{cases}$$

Equation of $f(x)$

Pictured above is the graph of a function $g(x)$ and the equation of a piece-wise defined function $f(x)$. Answer the following questions.

a. Find $\lim\limits_{x\to 1^+} \left[2g(x) - f(x)\cdot \cos \pi x\right]$. Show your work applying the properties of limits.

$$\left(\lim_{x\to 1^+} 2\right)\left(\lim_{x\to 1^+} g(x)\right) - \left(\lim_{x\to 1^+} f(x)\right)\left(\lim_{x\to 1^+} \cos \pi x\right)$$

$$= (2)(2) - (1^2 - 3(1))(\cos \pi)$$

$$= 4 - (-2)(-1)$$

$$= 4 - 2 = \boxed{2}$$

b. On its domain, what is one value of x at which $g(x)$ is discontinuous? Use the three part definition of continuity to explain why $g(x)$ is discontinuous at this value.

$g(x)$ IS DISCONTINUOUS AT $x = 1$:

① $g(1) = 0$ ($g(1)$ IS DEFINED)

② $\lim_{x \to 1^-} g(x) = 0$ $\lim_{x \to 1^+} g(x) = 2$, SO $\lim_{x \to 1^-} g(x) \neq \lim_{x \to 1^+} g(x)$

$\therefore$ $g(x)$ IS DISCONTINUOUS AT $x = 1$

c. For what value(s) of a and b, if they exist, would the function $f(x)$ be continuous everywhere? Justify your answer.

for a

$a(-3)+3 = (-3)^2 - 3(-3)$

$-3a + 3 = 18$

$-3a = 15$

$\boxed{a = -5}$

for b:

$2^2 - 3(2) = b(2) - 5$

$-2 = 2b - 5$

$3 = 2b$

$\boxed{\frac{3}{2} = b}$