Honors PreCalculus

Section 9.4: Sequences

What's a sequence?

- An ordered progression of numbers
  - Finite sequence: 2,4,6,8,10
  - Infinite sequence: 1, 3, 9, . . . , 3^k, . . .
  - Fibonacci sequence: 1, 1, 2, 3, 5, 8, 13, . . .

What's a sequence?

- Arithmetic sequence
  - Numbers in the sequence have a common difference
    - Add or subtract the same number each time
  - General "rule"
    - $a_n = a_1 + (n - 1)d$

nth term
1st term
Number of terms
Common difference

What's a sequence?

- Geometric sequence
  - Numbers in the sequence have a common ratio
    - Multiply or divide by the same number each time
  - General "rule"
    - $a_n = a_1(r^{n-1})$

OF TERMS

RATIO

n^{th} TERM

1^{st} TERM

Arithmetic Sequences

Example 1:

Find the common difference, the rule (equation) and the 10th term of the sequence: -3, -1, 1, 3, 5, . . .

- Common difference: Adding 2 each time, so d = 2
- Equation (rule): $a_n = a_1 + (n - 1)d$
  - $a_1 = -3$
  - $d = 2$
  - So: $a_n = -3 + (n - 1)(2) = -3 + 2n - 2 =$ 2n – 5
  - 10th term: n = 10, so $a_{10} = 2(10) - 5 = 20 - 5 = 15$

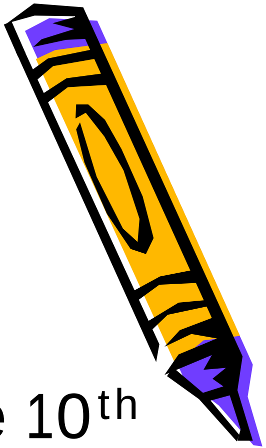
Arithmetic Sequences

Example 2:

Find the common difference, the rule (equation) and the 10th term of the sequence: 6, 2, -2, -6, -10, . . .

- Common difference: Subtracting 4 each time, so d = -4
- Equation (rule): $a_n = a_1 + (n – 1)d$
  - $a_1 = 6$
  - d = -4
  - So: $a_n = 6 + (n – 1)(-4) = 6 - 4n + 4 = -4n + 10$
  - 10th term: n = 10, so $a_{10} = -4(10) + 10 = -40 + 10 = -30$

Geometric Sequences



Example 3:

Find the common ratio, the rule (equation) and the 10th term of the sequence: 3, 6, 12, 24, 48, . . .

- Common ratio: Multiplying by 2 each time, so $r = 2$
- Equation (rule): $a_n = a_1(r^{n-1})$
  - $a_1 = 3$
  - $r = 2$
  - So: $a_n = 3(2^{n-1})$
  - 10th term: $n = 10$, so $a_{10} = 3(2^{10-1}) = 3(2^9) = 1536$

Constructing Sequences

Example 4:

If the 2nd and 5th terms of a sequence are 3 and 24, respectively, find the equation of the arithmetic sequence

$d = ?$ $a_1 = ?$

$a_2 = 3$

$a_5 = 24$

$$d = \frac{24-3}{5-2} = \frac{21}{3} = 7$$

$$a_1 = 3 - 7 = -4$$

so: $a_n = -4 + (n-1)7 = -4 + 7n - 7$

$= 7n - 11$

Constructing Sequences

Example 5:

If the 2^{nd} and 5^{th} terms of a sequence are 3 and 24, respectively, find the equation of the geometric sequence

r? a_1?

$a_2 = 3 \rightarrow 3 = a_1 r \Rightarrow a_1 = \frac{3}{r}$

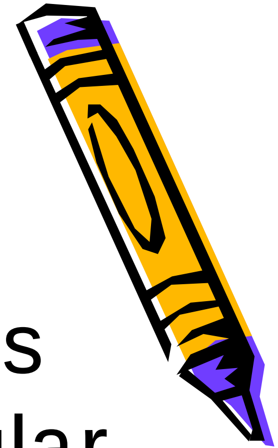
$a_5 = 24 \rightarrow 24 = a_1 r^4 \Rightarrow a_1 = \frac{24}{r^4}$

So… $\left(\frac{3}{r} = \frac{24}{r^4}\right) r^4$

$3r^3 = 24 \Rightarrow r^3 = 8 \Rightarrow r = 2$

$a_1 = \frac{3}{2}$ so $a_n = \frac{3}{2}\left(2^{n-1}\right)$

Limit of a sequence



A sequence converges if the numbers in the sequence approach a particular number

A sequence diverges if the numbers in the sequence approach infinity, or don’t actually approach any particular number

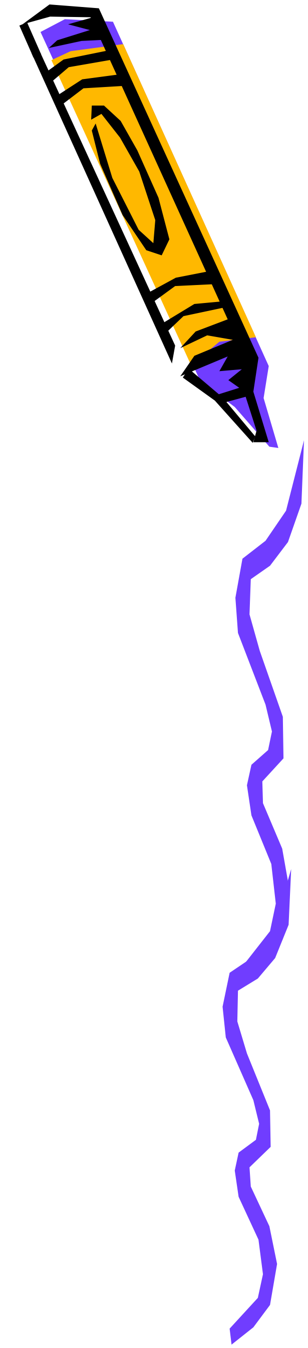
Limit of a sequence

Examples: $1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \cdots \frac{1}{n}, \cdots$ Converges to 0

$\frac{2}{1}, \frac{3}{2}, \frac{4}{3}, \frac{5}{4}, \cdots \frac{n+1}{n} = 1 + \frac{1}{n}$ Converges to 1

$2, 4, 6, 8, 10, \ldots$ Diverges

$-1, 1, -1, 1, \ldots, (-1)^n, \ldots$ Diverges



☺ Homework ☺

- Pages 739-740;
  - #12-30 (3s), 31, 32, 37, 40