Equation Solving and Modeling

Students will solve exponential equations that cannot be solved using One-to-One properties.

One-to-One Properties

For any exponential function $f(x) = b^x$

If $b^u = b^v$, then $u = v$

Example 1: Solve

a. $6(2^x) = 24$.

$2^x = 4$

$2^x = 2^2 \quad x = 2$

b. $1 + 5(3^{x-4}) = 46$

$5(3^{x-4}) = 45$

$3^{x-4} = 9 = 3^2$

$x - 4 = 2$

$x = 6$

If you cannot use the above method, you can use logarithms to solve an exponential equation.

1. **Isolate the exponential expression (if necessary).**
2. **Take the logarithm of both sides of the equation.**
3. **Use "basic" algebra to finish solving for the unknown.**

$\log_b b^x = x$

Example 2: Solve

a. $5^x = 134$

$\log_5 5^x = \log_5 134$

$x = \log_5 134 \approx 3.043$

$\frac{\log 134}{\log 5}$

b. $2.13^x = 6.4$

$\log_{2.13} 2.13^x = \log_{2.13} 6.4$

$x = \log_{2.13} 6.4$

c. $3(5^x) = 72$

$5^x = 24$

$x = \log_5 24$

d. $15 + 7(8^x) = 99$

$7(8^x) = 84$

$8^x = 12$

$x = \log_8 12$

e. $5e^x = 35$

$\frac{5}{5} = \frac{\ln e^x}{5} = \ln 7$

$x = \ln 7$

Your turn—solve the equations for x.

5. $4^x = 17$

$x = \log_4 17$

≈ 2.04

6. $5^{x-1} = 7$

$x - 1 = \log_5 7$

$x = (\log_5 7) + 1 \approx 2.21$

7. $15 + 3^x = 25$

$3^x = 10$

$x = \log_3 10 \approx 2.1$

8. $4 - 3(2^{x+5}) = -11$

$-3(2^{x+5}) = -15$

$2^{x+5} = 5$

$x + 5 = \log_2 5$

$x = (\log_2 5) - 5 \approx -2.68$

9. $4 + 7e^{3x} = 32$

$7e^{3x} = 28$

$e^{3x} = 4$

$3x = \ln 4$

$x = \frac{\ln 4}{3} \approx 0.46$