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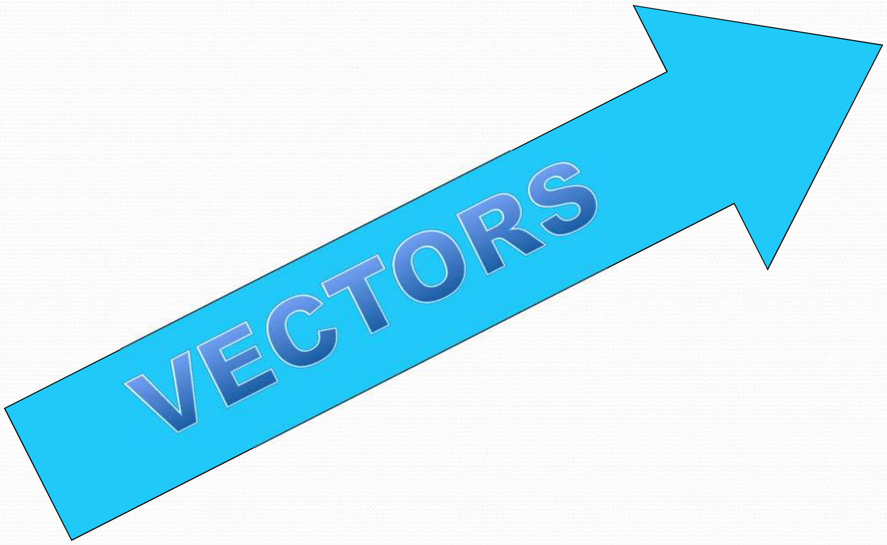
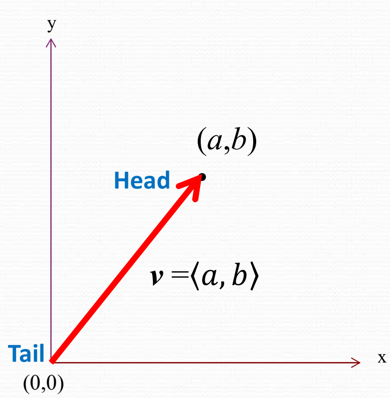


HPC/RPC 2017

Two – Dimensional Vectors



While **(a,b)** denotes a point on a plane it also represents a ***directed line segment*** with its tail (initial point) at the origin (0,0) and the head (terminal point) at **(a,b).**

The vector has both **magnitude** and **direction** and is denoted as $\langle \boldsymbol{a}, \boldsymbol{b} \rangle$ in order to distinguish from point **(a,b).**

<u>DEFINITION</u>: Two-Dimensional Vector

A 2-D vector, ***v***, is an ordered pair of real numbers in component form $\langle a, b\rangle$.

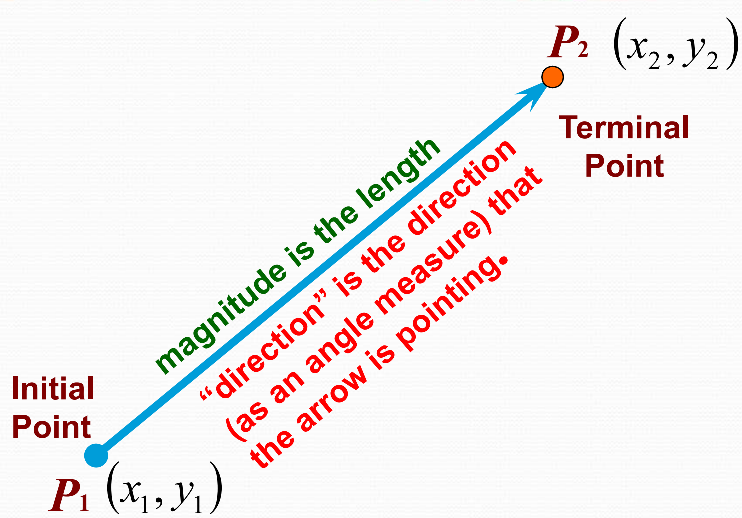
The numbers *a, b* are the **<u>components</u>** of the vector **v**.

The standard representation (also known as "**component form**") of vector $\langle a, b\rangle$ is the arrow from the origin (0,0) to point (a,b).

The **magnitude** of ***v*** is the length of the line segment.

The **direction** of ***v*** is the direction (given as an angle measure) that the arrow is pointing.

Zero vector, **0** = $\langle 0, 0\rangle$ has no length or direction



Component Form of a Vector

Vectors can be compared (magnitude and direction) as long as they are in **component form**.

Head Minus Tail (HMT) Rule to find Component Form:
If a vector has initial point (x_1, y_1) and terminal point (x_2, y_2), it represents the vector $\langle x_2 - x_1, y_2 - y_1 \rangle$

Example 1: Find component form of vectors **u** and **v**.
u has initial point (1, 1) and terminal point (2, 4)

$\langle 2 - 1, 4 - 1 \rangle = \langle 1, 3 \rangle$

v has initial point (5, 3) and terminal point (6, 6)

$\langle 6 - 5, 6 - 3 \rangle = \langle 1, 3 \rangle$

Magnitude

- **If v is represented by the arrow from (x_1, y_1) to (x_2, y_2) then the magnitude of the vector is represented by:**

$$\|v\| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

- **If the vector is in Component Form $\langle a, b \rangle$, the magnitude can be found using**

$$\|v\| = \sqrt{a^2 + b^2}$$

Magnitude

Example 1: Find the magnitude of vectors **u** and **v**.
u has initial point (1, 1) and terminal point (2, 4)

$$\|u\| = \sqrt{(2-1)^2+(4-1)^2}$$
$$= \sqrt{1^2+3^2}$$
$$= \sqrt{1+9} \qquad = \sqrt{\mathbf{10}}$$

v has initial point (5, 3) and terminal point (6, 6)

$$\|v\| = \sqrt{(6-5)^2+(6-3)^2}$$
$$= \sqrt{1^2+3^2}$$
$$= \sqrt{1+9} \qquad = \sqrt{\mathbf{10}}$$

Vector Operations

Vector Addition

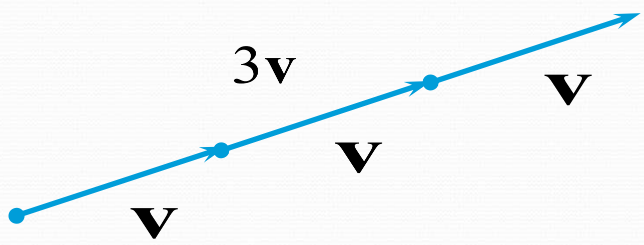
if ***u*** = $\langle u_1, u_2 \rangle$ and
v = $\langle v_1, v_2 \rangle$,
the **sum** of vectors ***u*** and ***v*** is the vector:
u + ***v*** = $\langle u_1 + v_1, u_2 + v_2 \rangle$
(This is also know as the "resultant vector")

Scalar Multiplication

if ***u*** = $\langle u_1, u_2 \rangle$ and
k is a real number (scalar) the product of the scalar *k* and the vector ***u*** is:
*k**u*** = $k\langle \boldsymbol{u_1}, \boldsymbol{u_2} \rangle = \langle k\boldsymbol{u_1}, k\boldsymbol{u_2} \rangle$

The negative of a vector is just a vector going the opposite way.

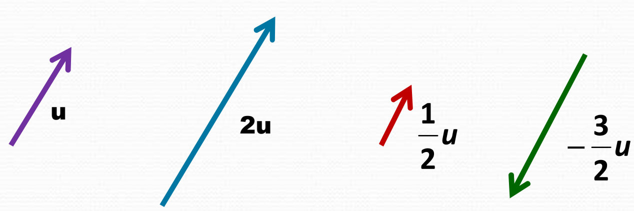
A number multiplied in front of a vector is called a scalar. It means to take the vector and add together that many times.



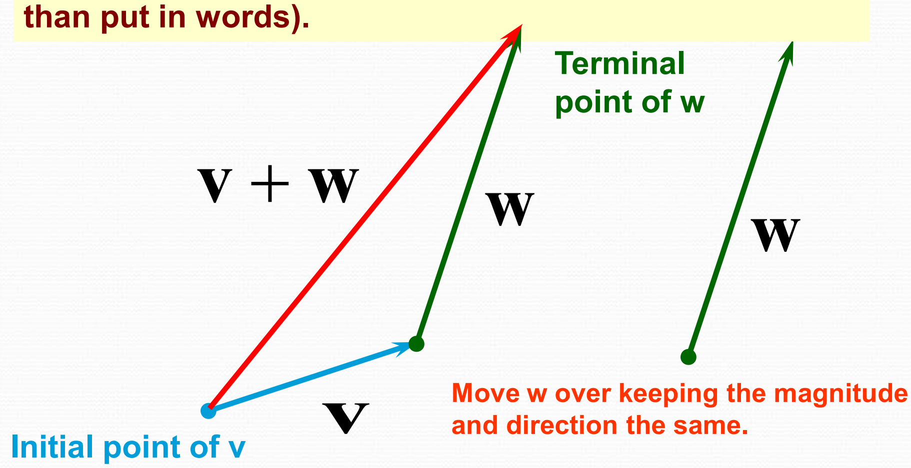
Scalar Multiplication

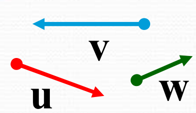
A vector can be multiplied by a real number k.

- If k is a positive number then the magnitude of the vector is changed
- If k is a negative number then the magnitude is changed and the direction is reversed.

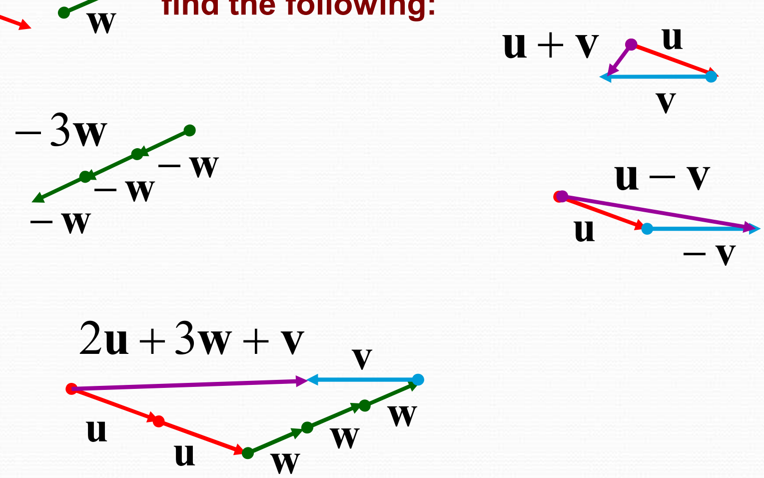


To add vectors, we put the initial point of the second vector on the terminal point of the first vector. The *resultant vector has an initial point at the initial point of the first vector and a terminal point at the terminal point of the second vector* (see below--better shown than put in words).





Using the vectors shown, find the following:



Example 3: Performing Vector Operations

If $\boldsymbol{u} = \langle -3, 2 \rangle$ and $\mathbf{v} = \langle 7, 3 \rangle$ find the following:

a. $\boldsymbol{u} + \boldsymbol{v}$ = <-3 + 7, 2 + 3> = <4, 5>

b. $3\boldsymbol{v}$ = 3 <7, 3> = <21, 9>

c. $2\boldsymbol{u} + (-1)\boldsymbol{v}$ =2 <-3, 2> + (-1) <7, 3>

=<-6, 4> + <-7, -3>

= <-13, 1>