Page 255: #15 - 33 (odds)

15. $\int (x + 7)\,dx = \frac{x^2}{2} + 7x + C$

17. $\int (2x - 3x^2)\,dx = \frac{2x^2}{2} - \frac{3x^3}{3} + C$
$= x^2 - x^3 + C$

19. $\int (x^5 + 1)\,dx = \frac{x^6}{6} + \frac{x^1}{1} + C$
$= \frac{x^6}{6} + x + C$

21. $\int (x^{3/2} + 2x + 1)\,dx = \frac{x^{5/2}}{5/2} + \frac{2x^2}{2} + x + C$
$= \frac{2}{5}x^{5/2} + x^2 + x + C$

23. $\int \sqrt[3]{x^2}\,dx = \int x^{2/3}\,dx$
$= \frac{x^{5/3}}{5/3} + C = \frac{3}{5}x^{5/3} + C$

25. $\int \frac{1}{x^5}\,dx = \int x^{-5}\,dx$
$= \frac{x^{-4}}{-4} + C = -\frac{1}{4x^4} + C$

27. $\int \frac{x + 6}{\sqrt{x}}\,dx = \int x^{-1/2}(x+6)\,dx$
$= \int (x^{1/2} + 6x^{-1/2})\,dx = \frac{x^{3/2}}{3/2} + \frac{6x^{1/2}}{1/2} + C$
$= \frac{2}{3}x^{3/2} + 12x^{1/2} + C$

29. $\int (x + 1)(3x - 2)\,dx = \int (3x^2 + x - 2)\,dx$
$= \frac{3x^3}{3} + \frac{x^2}{2} - 2x + C = x^3 + \frac{x^2}{2} - 2x + C$

31. $\int y^2\sqrt{y}\,dy = \int (y^2 \cdot y^{1/2})\,dy$
$= \int y^{5/2}\,dy = \frac{y^{7/2}}{7/2} + C = \frac{2}{7}y^{7/2}$

33. $\int dx = \int 1\,dx = x + C$

Page 258: #81

Rectilinear Motion **In Exercises 81–84, consider a particle moving along the x-axis where $x(t)$ is the position of the particle at time t, $x'(t)$ is its velocity, and $x''(t)$ is its acceleration.**

81. $x(t) = t^3 - 6t^2 + 9t - 2, \quad 0 \le t \le 5$

(a) Find the velocity and acceleration of the particle.

(b) Find the open t-intervals on which the particle is moving to the right. $v(t) > 0$

(c) Find the velocity of the particle when the acceleration is 0.

(a) $v(t) = x'(t) = 3t^2 - 12t + 9$

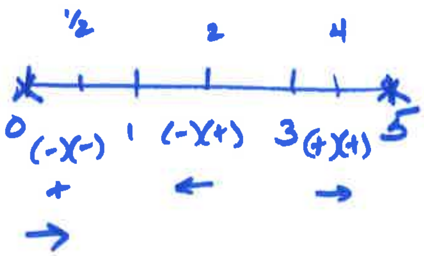
$a(t) = v'(t) = x''(t) = 6t - 12$

(b) $v(t) = 3t^2 - 12t + 9 = 0$

$t^2 - 4t + 3 = 0$

$(t-3)(t-1) = 0$

$\underbrace{t = 3 \quad t = 1}_{\text{Critical Points}}$



MOVING RIGHT ON $(0,1) \cup (3,5)$

(c) $a(t) = 6t - 12 = 0$

$t - 2 = 0$

$t = 2$

$\cancel{v(2) = (2-3)(2-1) = (-)(+) = (-)}$

$v(2) = 3(2)^2 - 12(2) + 9$

$= 12 - 24 + 9 = -3$

Page 279: #33, 35, 37, 43, 47

In Exercises 33–40, evaluate the integral using the following values.

$$\int_2^4 x^3\,dx = 60, \qquad \int_2^4 x\,dx = 6, \qquad \int_2^4 dx = 2$$

33. $\int_4^2 x\,dx = -\int_2^4 x\,dx = -6$

35. $\int_2^4 8x\,dx = 8\int_2^4 x\,dx = 8(6) = 48$

37. $\int_2^4 (x-9)\,dx = \int_2^4 x\,dx - \int_2^4 9\,dx = 6 - 9x\Big|_2^4 = 6-(9(4)-9(2))$
$= 6-(36-18) = 6-18 = -12$

43. Given $\int_2^6 f(x)\,dx = 10$ and $\int_2^6 g(x)\,dx = -2$, evaluate

(a) $\int_2^6 [f(x) + g(x)]\,dx.$ (b) $\int_2^6 [g(x) - f(x)]\,dx.$

(c) $\int_2^6 2g(x)\,dx.$ (d) $\int_2^6 3f(x)\,dx.$

(a) $\int_2^6 f(x)\,dx + \int_2^6 g(x)\,dx = 10+(-2) = 8$

(b) $\int_2^6 g(x)\,dx - \int_2^6 f(x)\,dx = -2-10 = -12$

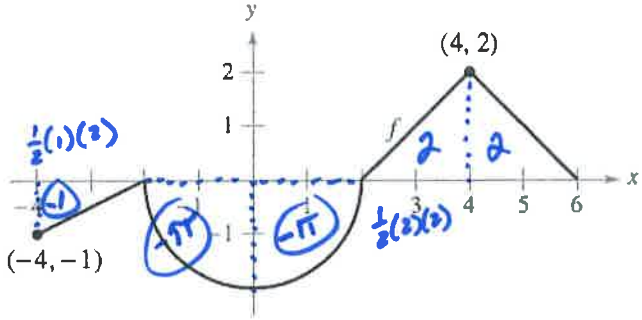
(c) $2\int_2^6 g(x)\,dx = 2(-2) = -4$

(d) $3\int_2^6 f(x)\,dx = 3(10) = 30$

Page 279: #33, 35, 37, 43, 47

4π

47. ***Think About It*** The graph of f consists of line segments and a semicircle, as shown in the figure. Evaluate each definite integral by using geometric formulas.



(a) $\int_0^2 f(x)\,dx$ (b) $\int_2^6 f(x)\,dx$ (c) $\int_{-4}^2 f(x)\,dx$

(d) $\int_{-4}^6 f(x)\,dx$ (e) $\int_{-4}^6 |f(x)|\,dx$ (f) $\int_{-4}^6 [f(x)+2]\,dx$

(a) $\int_0^2 f(x)\,dx = -\pi$

(b) $\int_2^6 f(x)\,dx = 2+2 = 4$

(c) $\int_{-4}^2 f(x)\,dx = -1 + (-\pi) + (-\pi) = -1-2\pi$

(d) $\int_{-4}^6 f(x)\,dx = -1 + (-\pi) + (-\pi) + 2 + 2 = 3-2\pi$

(e) $\int_{-4}^6 |f(x)|\,dx = 1 + \pi + \pi + 2 + 2 = 5+2\pi$

* (f) $\int_{-4}^6 [f(x)+2]\,dx = \int_{-4}^6 f(x)\,dx + \int_{-4}^6 2\,dx = 3-2\pi + 2x\Big|_{-4}^{6}$

$= 3-2\pi+2(6)-2(-4)$

$= 3-2\pi+12+8 = \cancel{35-2\pi}$

$23-2\pi$

Page 296: #103, 105

103. A particle is moving along the x-axis. The position of the particle at time t is given by $x(t) = t^3 - 6t^2 + 9t - 2$, $0 \le t \le 5$. Find the total distance the particle travels in 5 units of time.

$$v(t) = x'(t) = 3t^2 - 12t + 9$$

TOTAL DIST. $\int_0^5 |v(t)|\,dt = \int_0^5 |3t^2 - 12t + 9|\,dt = 28$

105. ***Water Flow*** Water flows from a storage tank at a rate of $500 - 5t$ liters per minute. Find the amount of water that flows out of the tank during the first 18 minutes.

$$\int_0^{18} (500 - 5t)\,dt = 500t\Big|_0^{18} - \frac{5t^2}{2}\Big|_0^{18}$$

$$= 500(18) - \frac{5(18)^2}{2}$$

$$= 8190 \text{ L}$$

INTERVALS:

$x = 1, 1.5, 2, 2.5, 3$: WIDTH $= 0.5$

Page AP 4-1: #1, 2

1. Define $f(x) = x^3 - 3x^2 + 3x$.
   (a) Estimate the area between the graph of f and the x-axis on the interval [1,3] using a left-hand sum with four rectangles of equal width.
   (b) Is the estimate in part (a) an overestimate or an underestimate of the actual area? Justify your conclusion.
   (c) Use a definite integral to calculate the exact area between the graph of f and the x-axis on the interval [1, 3].

(a) $A \approx 0.5(f(1) + f(1.5) + f(2) + f(2.5)) = 0.5(1 + 1.125 + 2 + 4.375)$

$= 0.5(8.5) \approx 4.25$

(b) SINCE $f(x)$ IS CONCAVE UP ON $[1,3]$, (a) IS AN UNDERESTIMATE

(c) $\int_1^3 (x^3 - 3x^2 + 3x)\,dx = \frac{x^4}{4}\Big|_1^3 - \frac{\cancel{3}x^3}{\cancel{3}}\Big|_1^3 + \frac{3x^2}{2}\Big|_1^3$

$= \frac{3^4}{4} - \frac{1^4}{4} - [3^3 - 1^3] + \left[\frac{3(3)^2}{2} - \frac{3(1)^2}{2}\right] = 20 - 26 + 12 = 6$

2. Define $f(x) = -x^2 + 4x$.
   (a) Estimate the area between the graph of f and the x-axis on the interval [1, 4] using a left-hand sum with four rectangles of equal width.
   (b) Estimate the area between the graph of f and the x-axis on the interval [1, 4] using a right-hand sum with four rectangles of equal width.
   (c) Use a definite integral to calculate the exact area between the graph of f and the x-axis on the interval [1, 4].

WIDTH $= 0.75$

(a) $0.75(f(1) + f(1.75) + f(2.5) + f(3.25))$

$= 0.75(3 + 3.938 + 3.75 + 2.438) = 0.75(12.586)$

≈ 9.440

(b) $0.75(f(1.75) + f(2.5) + f(3.25) + f(4))$

$0.75(3.938 + 3.75 + 2.438 + 0) = 0.75(9.586)$

≈ 7.190

(c) $\int_1^4 (-x^2 + 4x)\,dx = -\frac{x^3}{3}\Big|_1^4 + \frac{{}^2\cancel{4}x^2}{\cancel{2}}\Big|_1^4$

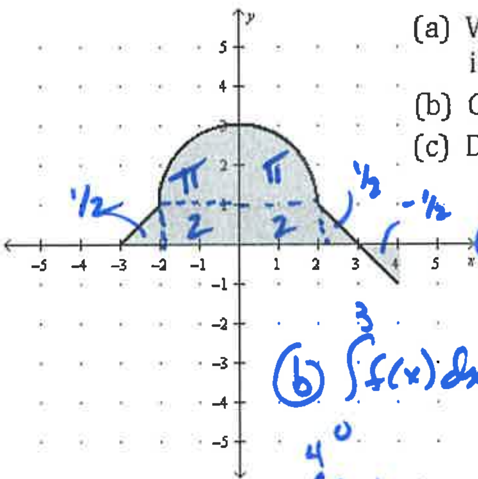
$= -\frac{4^3}{3} - \left(-\frac{1^3}{3}\right) + 2(4)^2 - 2(1)^2 = -\frac{64}{3} + \frac{1}{3} + 32 - 2$

$= -21 + 30 = 9$

$$f(x)\begin{cases} x+3; & -3<x<-2 \\ \sqrt{4-x^2}+1; & -2<x<2 \\ -x+3; & 2<x<4 \end{cases}$$

Page AP 4-1: #4, 5

4. The graph of a function f is shown in the figure below. It consists of two lines and a semicircle. The regions between the graph of f and the x-axis are shaded.



(a) Write the definite integral or the sum of definite integrals that measures the area of the shaded region.

(b) Calculate $\int_0^3 f(x)dx$ and $\int_3^4 f(x)dx$.

(c) Determine the area of the shaded region.

ⓐ $\int_{-3}^{-2}(x+3)dx + \int_{-2}^{2}\left(\sqrt{4-x^2}+1\right)dx + \int_{2}^{4}(-x+3)dx$

ⓑ $\int_0^3 f(x)dx = \int_0^2 \sqrt{4-x^2}+1 + \int_2^3(-x+3)dx \approx 5.641$

$\int_3^4 f(x)dx = \int_3^4(-x+3)dx = -0.5$

ⓒ $\int_{-3}^{4} f(x)dx = \frac{1}{2} + 2\pi + 4 + \cancel{\frac{1}{2} - \frac{1}{2}} = 4.5 + 2\pi$

5. Define $f(x) = 4x^3 - 4x$.

(a) Calculate $\int_0^1 f(x)dx$.

(b) What is the average value of f on $[0, 2]$?

(c) What is the area of the region(s) bounded by the graph of f and the x-axis? Show the work that leads to your conclusion.

ⓐ $\int_0^1(4x^3-4x)dx = \frac{\cancel{4}x^4}{\cancel{4}}\Big|_0^1 - \frac{2\cancel{4}x^2}{\cancel{2}}\Big|_0^1 = 1^4 - 2(1)^2 = -1$

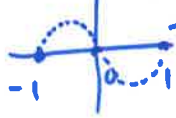
ⓑ $\frac{1}{2-0}\int_0^2(4x^3-4x)dx = \frac{1}{2}\left[\frac{\cancel{4}x^4}{\cancel{4}} - \frac{2\cancel{4}x^2}{\cancel{2}}\right]_0^2 = \frac{1}{2}\left[2^4 - 2(2)^2\right] = 4$

ⓒ $4x^3 - 4x = 0$

$4x(x^2-1) = 0$

$\underbrace{x=0 \quad x=1 \quad x=-1}_{\text{CRITICAL POINTS}}$

GRAPH OF $f(x)$



$\int_{-1}^{1}(4x^3-4x)dx = x^4\Big|_{-1}^{1} - 2x^2\Big|_{-1}^{1}$

$= 1^4 - (-1)^4 - \left[2(1)^2 - 2(-1)^2\right]$

$= 1 - 1 - 2 + 2 = 0$