HPC/RPC Review Name_____ Period_____

Properties of Exponentials and Logarithms
and Solving Exponential and Logarithmic equations

Write the equation in its equivalent exponential form.

1) $\log_4 16 = 2$

$4^2 = 16$

2) $\log_b 64 = 2$

$b^2 = 64$

Write the equation in its equivalent logarithmic form.

3) $6^3 = 216$

$\log_6 216 = 3$

4) $\sqrt[3]{343} = 7$ $\rightarrow$ $343^{1/3} = 7$ $\rightarrow$ $\log_{343} 7 = \frac{1}{3}$

Solve the equation by expressing each side as a power of the same base and then equating exponents.

5) $4^x = 16$

$4^x = 4^2$ $\quad x = 2$

6) $3^{(3x+6)} = \frac{1}{27}$

$3^{3x+6} = 3^{-3}$ $\rightarrow$ $3x+6 = -3$

$3x = -9$

$x = -3$

7) $1024^x = \frac{1}{\sqrt{4}}$ $\rightarrow (2^{10})^x = 2^{-1} \rightarrow 2^{10x} = 2^{-1}$ $\quad 10x = -1$

$\downarrow$ $\quad x = -1/10$

$\frac{1}{4^{1/2}} \rightarrow \frac{1}{2}$

Solve the exponential equation. Use a calculator to obtain a decimal approximation, correct to two decimal places, for the solution.

8) $10^x = 3.19$ $\quad \cancel{\log 10}^x = \log 3.19$

$x = \log 3.19 \approx 0.50$

9) $e^x = 4.2$ $\quad \cancel{\ln e}^x = \ln 4.2$

$x = \ln 4.2 \approx 1.44$

10) $e^{3x} = 5$ $\quad \cancel{\ln e}^{3x} = \ln 5$

$3x = \ln 5$

$x = \frac{\ln 5}{3} \approx 0.54$

11) $e^{4x-5} - 2 = 1182$ $\quad e^{4x-5} = 1180$

$\cancel{\ln e}^{4x-5} = \ln 1180$

$4x - 5 = \ln 1180$

$4x = (\ln 1180) + 5$

$x = \frac{(\ln 1180) + 5}{4} \approx 3.02$

Solve the logarithmic equation. Be sure to reject any value that is not in the domain of the original logarithmic expressions. Give the exact answer.

12) $\log_3 x = 4$

$3^4 = x \rightarrow x = 81$

13) $\log_4 (x + 3) + \log_4 (x - 3) = 2$

$\log_4 (x+3)(x-3) = 2 \rightarrow 4^2 = x^2 - 9 \rightarrow 16 = x^2 - 9 \rightarrow x^2 = 25$

$x = \pm 5$

$x = 5$

$x = -5$ NOT A SOLUTION

14) $4 + 8 \ln x = 8$

$8 \ln x = 4$

$\ln x = \frac{4}{8} = \frac{1}{2}$

$x = e^{\frac{1}{2}} \approx 1.65$

15) $\log_3 (x + 6) + \log_3 (x - 6) - \log_3 x = 2$

$\log_3 \frac{(x+6)(x-6)}{x} = 2$

$3^2 = \frac{(x+6)(x-6)}{x}$

$9 = \frac{x^2-36}{x}$

$9x = x^2 - 36$

$0 = x^2 - 9x - 36$

$0 = (x-12)(x+3)$

$x = 12$ $\cancel{x = -3}$

16) $\log_2 (x + 3) = 4 + \log_2 (x - 2)$

$\log_2(x+3) - \log_2(x-2) = 4$

$\log_2\left(\frac{x+3}{x-2}\right) = 4$

$2^4 = \frac{x+3}{x-2}$

$16 = \frac{x+3}{x-2}$

$16(x-2) = x+3$

$16x - 32 = x + 3$

$15x = 35$

$x = \frac{35}{15} = \frac{7}{3}$

$x \approx 2.33$

17) $\log (3 + x) - \log (x - 2) = \log 2$

$\log \frac{3+x}{x-2} = \log 2$

$\frac{3+x}{x-2} = 2$

$3 + x = 2(x-2)$

$3 + x = 2x - 4$

$7 = x$

18) $\log 3x = \log 4 + \log (x - 3)$

$\log 3x = \log 4(x-3)$

$3x = 4x - 12$

$-x = -12 \rightarrow x = 12$

19) $\ln x + \ln (x + 1) = \ln 6$

$\ln x(x+1) = \ln 6$

$x(x+1) = 6$

$x^2 + x = 6$

$x^2 + x - 6 = 0$

$(x+3)(x-2) = 0$

$\cancel{x = -3}$ $x = 2$

20) $\log (x + 20) - \log 2 = \log (3x + 4)$

$\log \frac{x+20}{2} = \log(3x+4)$

$\frac{x+20}{2} = 3x + 4$

$x + 20 = 6x + 8$

$12 = 5x$

$x = \frac{12}{5} = 2.4$

21) $\ln (x - 8) - \ln (x + 7) = \ln (x - 10) - \ln (x + 8)$

$\ln\left(\frac{x-8}{x+7}\right) = \ln\left(\frac{x-10}{x+8}\right)$

$\frac{x-8}{x+7} = \frac{x-10}{x+8}$

$(x-8)(x+8) = (x+7)(x-10)$

$\cancel{x^2} - 64 = \cancel{x^2} - 3x - 70$

$-64 = -3x - 70$

$6 = -3x$

$\cancel{x = -2}$

NO SOLUTION