Name________________________________ Date__________________ Class__________

Day #45 Homework

1. Find the point on the graph of $f(x)=\sqrt{-x+8}$ so that the point (2, 0) is closest to the graph.

MINIMIZE DISTANCE

$d=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

$=\sqrt{(x-2)^2+(0-\sqrt{-x+8})^2}$

$=\sqrt{x^2-4x+4+(-x)+8}$

$=\sqrt{x^2-5x+12}$

$d=(x^2-5x+12)^{1/2}$

$d'=\frac{1}{2}(x^2-5x+12)^{-1/2}(2x-5)$

$=\dfrac{2x-5}{2\sqrt{x^2-5x+12}}=0$

$2x-5=0$

$x=5/2$

1   3

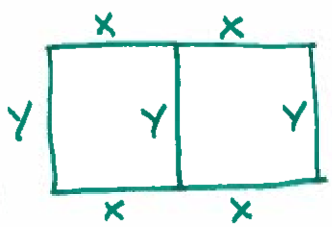
−   5/2   +   +

MINIMUM DISTANCE AT $\left(\frac{5}{2},\sqrt{\frac{11}{2}}\right)$

$f(5/2)=\sqrt{-\frac{5}{2}+8\frac{16}{2}}$

$=\sqrt{\frac{11}{2}}$

2. A rancher has 200 total feet of fencing with which to enclose two adjacent rectangular corrals. What dimensions should each corral be so that the enclosed area will be a maximum? MAXIMIZE AREA



$A=2xy$

$200=4x+3y$

$3y=200-4x \rightarrow y=\dfrac{200-4x}{3}$

$A=2x\left(\dfrac{200-4x}{3}\right)$

$=\dfrac{400x-8x^2}{3}$

$=\dfrac{400}{3}x-\dfrac{8}{3}x^2$

$A'=\dfrac{400}{3}-\dfrac{16}{3}x=0$

$\dfrac{3}{16}\cdot\dfrac{400}{3}=\dfrac{16}{3}x$

$25=x$

20   30

+   25   −

$200=4(25)+3y$

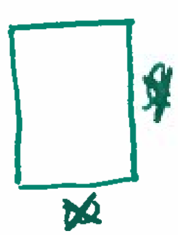
$100=3y$

$\dfrac{100}{3}=y$

$x=25$ ft

$y=\dfrac{100}{3}$ ft

3. The area of a rectangle is 64 square feet. What dimensions of the rectangle would give the smallest perimeter?



$xy=64$ $y=\dfrac{64}{x}$

$P=2x+2y$

$P=2x+2\left(\dfrac{64}{x}\right)$

$=2x+\dfrac{128}{x}$

$P=2x+128x^{-1}$

$P'=2-128x^{-2}$

$=2-\dfrac{128}{x^2}=0$

$\dfrac{128}{x^2}=2$

$128=2x^2$

$64=x^2$

$8=x$

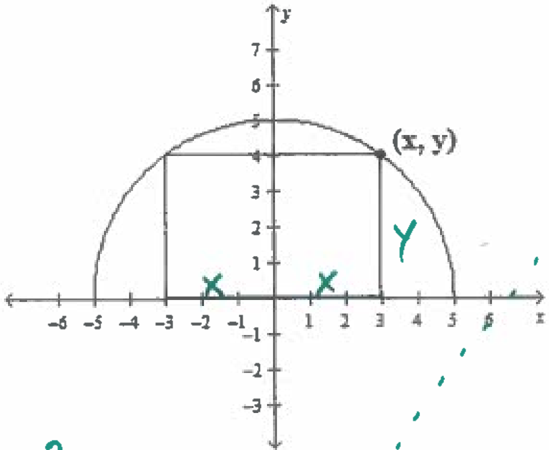
1   10

−   8   +

$x=8$

$y=8$

4. A rectangle is bound by the x – axis and the graph of a semicircle defined by $y=\sqrt{25-x^2}$. What length and width should the rectangle have so that its area is a maximum?



$A = 2xy$

$A = 2x(\sqrt{25-x^2})$

$A = 2x(25-x^2)^{1/2}$

$A' = 2(25-x^2)^{1/2} + 2x\left(\frac{1}{2}(25-x^2)^{-1/2}(-2x)\right)$

$= 2(25-x^2)^{1/2} - 2x^2(25-x^2)^{-1/2}$

$= 2\sqrt{25-x^2} - \frac{2x^2}{\sqrt{25-x^2}}$

$= \frac{2(25-x^2)-2x^2}{\sqrt{25-x^2}} = \frac{50-2x^2-2x^2}{\sqrt{25-x^2}}$

$= \frac{50-4x^2}{\sqrt{25-x^2}} = 0$

$50-4x^2=0$

$50=4x^2$

$\frac{25}{2}=x^2$

$\frac{5}{\sqrt{2}}=x$

1 4

$+$ $\frac{5}{\sqrt{2}}$ $-$

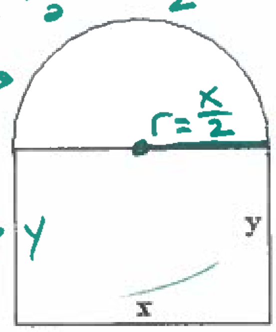
3.53

$y=\sqrt{25-\left(\frac{5}{\sqrt{2}}\right)^2} = \sqrt{25-\frac{25}{2}} = \sqrt{\frac{25}{2}}$

AREA IS MAX WHEN

$x=\frac{5}{\sqrt{2}}$ $y=\frac{5}{\sqrt{2}}$

5. A Norman window is constructed by adjoining a semicircle to the top of an ordinary rectangular window (see figure below). Find the dimensions of a Norman window of maximum area if the total perimeter is 16 feet.



$A=\frac{\pi r^2}{2} = \frac{\pi x^2}{8}$

$\frac{\pi(\frac{x}{2})^2}{2} = \frac{\frac{\pi x^2}{4}}{2}$

$r=\frac{x}{2}$

$A=xy$

$A=\frac{\pi x^2}{8}+xy$

$P = x+2y+\frac{\pi x}{2}=16$

$2y = 16 - x - \frac{\pi x}{2}$

$y = 8-\frac{x}{2}-\frac{\pi x}{4}$

$A = \frac{\pi x^2}{8} + x\left(8-\frac{x}{2}-\frac{\pi x}{4}\right)$

$= \frac{\pi x^2}{8}+8x-\frac{x^2}{2}-\frac{\pi x^2}{4}$

$A = \frac{\pi}{8}x^2+8x-\frac{1}{2}x^2-\frac{\pi}{4}x^2$

$A' = \frac{\pi}{4}x+8-x-\frac{\pi}{2}x$

$= 8-x-\frac{\pi}{4}x=0$

$8=x\left(1+\frac{\pi}{4}\right)$

$x=\frac{8}{1+\frac{\pi}{4}}$

$x=4.481$

4 5

$+$ 4.481 $-$

$y=8-\frac{4.481}{2}-\frac{\pi(4.481)}{4}$

$y=2.240$

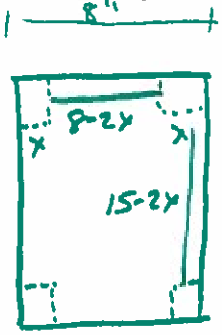
MAX AREA WHEN

$x=4.481$ ft

$y=2.240$ ft

$-30 - 16$

6. Find the maximum volume of a box that can be made by cutting squares from the corners of an 8 inch by 15 inch rectangular sheet of cardboard and folding up the sides.



$$V = x(8-2x)(15-2x)$$
$$= x(120 - 46x + 4x^2)$$
$$V = 4x^3 - 46x^2 + 120x$$
$$V' = \frac{12}{4}x^2 - \frac{92}{4}x + \frac{120}{4} = 0$$
$$3x^2 - 23x + 30 = 0$$
$$(3x - 5)(x - 6) = 0$$
$$x = 5/3 \qquad \cancel{x = 6}$$

1 2

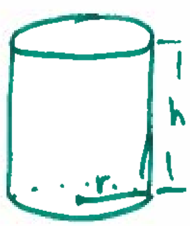
$\frac{5}{3}$

$+ \quad -$

$x = 5/3$ in GIVES THE MAXIMUM VOLUME

$$V = 4\left(\tfrac{5}{3}\right)^3 - 46\left(\tfrac{5}{3}\right)^2 + 120\left(\tfrac{5}{3}\right)$$
$$V = 90.741 \text{ in}^3$$

7. The volume of a cylindrical tin can with a top and bottom is to be 16π cubic inches. If a minimum amount of tin is to be used to construct the can, what much the height, in inches, of the can be?



$$V = \pi r^2 h = 16\pi$$
$$SA = 2\pi r^2 + 2\pi r h$$
$$16\pi = \pi r^2 h$$
$$\cancel{\frac{16\pi}{\pi h} = r^2}$$
$$h = \frac{16\pi}{\pi r^2} = \frac{16}{r^2}$$

$$SA = 2\pi r^2 + 2\pi r\left(\frac{16}{r^2}\right)$$
$$= 2\pi r^2 + 32\pi r^{-1}$$
$$SA' = 4\pi r - 32\pi r^{-2}$$
$$= 4\pi r - \frac{32\pi}{r^2} = 0$$
$$4\pi r = \frac{32\pi}{r^2}$$
$$4\pi r^3 = 32\pi$$
$$r^3 = 8$$
$$r = 2$$

1 3

A' $- \quad 2 \quad +$

$4\pi - 32\pi \qquad 12\pi - \frac{32}{9}\pi$

MIN. HEIGHT WHEN $r = 2$

$$h = \frac{16}{2^2} = \frac{16}{4} = 4 \text{ in}$$