

Day #12 Homework

Find $\lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$ for each of the functions below. Then, find the equation of the tangent line to the graph of $f(x)$ at the given value of x .

<p>1. $f(x) = x^3 + 2x$</p> $\lim_{h \rightarrow 0} \frac{(x+h)^3 + 2(x+h) - (x^3 + 2x)}{h}$ $\lim_{h \rightarrow 0} \frac{(x+h)(x^2 + 2xh + h^2) + 2x + 2h - x^3 - 2x}{h}$ $\lim_{h \rightarrow 0} \frac{x^3 + 2x^2h + xh^2 + x^3h + 2xh^2 + h^3 + 2hx^3}{h}$ $\lim_{h \rightarrow 0} \frac{3x^2h + 3xh^2 + h^3 + 2h}{h}$ $\lim_{h \rightarrow 0} \frac{h(3x^2 + 3xh + h^2 + 2)}{h}$ $3x^2 + 3x(0) + (0)^2 + 2$ $3x^2 + 2$ <div style="border: 1px solid black; padding: 5px; width: fit-content; margin: 0 auto;"> $f'(x) = 3x^2 + 2$ </div>	<p>3. $f(x) = \sqrt{3-x}$</p> $\lim_{h \rightarrow 0} \frac{\sqrt{3-(x+h)} - \sqrt{3-x}}{h}$ $\lim_{h \rightarrow 0} \frac{\sqrt{3-x-h} - \sqrt{3-x}}{h} \cdot \frac{\sqrt{3-x-h} + \sqrt{3-x}}{\sqrt{3-x-h} + \sqrt{3-x}}$ $\lim_{h \rightarrow 0} \frac{3-x-h - (3-x)}{h(\sqrt{3-x-h} + \sqrt{3-x})}$ $\lim_{h \rightarrow 0} \frac{3-x-h - 3+x}{h(\sqrt{3-x-h} + \sqrt{3-x})}$ $\lim_{h \rightarrow 0} \frac{-1}{\sqrt{3-x-h} + \sqrt{3-x}}$ $\frac{-1}{\sqrt{3-x-0} + \sqrt{3-x}}$ <div style="border: 1px solid black; padding: 5px; width: fit-content; margin: 0 auto;"> $f'(x) = \frac{-1}{2\sqrt{3-x}}$ </div>
<p>2. Find the equation of the line tangent to the graph of $f(x) = x^3 + 2x$ at $x = -1$.</p> <p>P.O.T: $f(-1) = (-1)^3 + 2(-1)$ $= -1 - 2 = -3$</p> <p>S.O.T: $f'(-1) = 3(-1)^2 + 2$ $= 3 + 2 = 5$</p> <div style="border: 1px solid black; padding: 5px; width: fit-content; margin: 0 auto;"> $y + 3 = 5(x + 1)$ </div>	<p>4. Find the equation of the line tangent to the graph of $f(x) = \sqrt{3-x}$ at $x = -6$.</p> <p>P.O.T: $f(-6) = \sqrt{3 - (-6)} = \sqrt{9} = 3$</p> <p>S.O.T: $f'(-6) = \frac{-1}{2\sqrt{3 - (-6)}}$ $= \frac{-1}{2(3)} = -\frac{1}{6}$</p> <div style="border: 1px solid black; padding: 5px; width: fit-content; margin: 0 auto;"> $y - 3 = -\frac{1}{6}(x + 6)$ </div>

For problems 5 - 9, use the function $f(x) = \frac{x}{x+2}$.

5. Find $f'(x)$ by finding $\lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$.

$$\lim_{h \rightarrow 0} \frac{\frac{x+h}{x+h+2} - \frac{x}{x+2}}{h}$$

$$\lim_{h \rightarrow 0} \frac{(x+h)(x+2) - x(x+h+2)}{(x+h+2)(x+2)} \cdot \frac{1}{h}$$

$$\lim_{h \rightarrow 0} \frac{x^2 + 2x + xh + 2h - x^2 - xh - 2x}{h(x+h+2)(x+2)}$$

$$\lim_{h \rightarrow 0} \frac{h(x+2-h)}{h(x+h+2)(x+2)}$$

$$\lim_{h \rightarrow 0} \frac{2}{(x+h+2)(x+2)}$$

$$\frac{2}{(x+0+2)(x+2)}$$

$$f'(x) = \frac{2}{(x+2)^2}$$

6. Find the slope of the tangent line drawn to the graph of $f(x)$ at $x = -2$.

$$f'(-2) = \frac{2}{(-2+2)^2} = \frac{2}{0} = \text{undefined}$$

7. Find the slope of the tangent line drawn to the graph of $f(x)$ at $x = -1$.

$$f'(-1) = \frac{2}{(-1+2)^2} = \frac{2}{1} = 2$$

8. Find the equation of the tangent line drawn to the graph of $f(x)$ at $x = -1$.

$$f(-1) = \frac{-1}{-1+2} = \frac{-1}{1} = -1$$

$$f'(-1) = 2$$

$$y + 1 = 2(x + 1)$$

9. Find $\lim_{x \rightarrow a} \frac{f(x) - f(a)}{x - a}$, where $a = -1$.

$$\lim_{x \rightarrow -1} \frac{\frac{x}{x+2} - (-1)}{x+1}$$

$$\lim_{x \rightarrow -1} \frac{\frac{x}{x+2} + \frac{x+2}{x+2}}{x+1}$$

$$\lim_{x \rightarrow -1} \frac{2x+2}{x+2} \cdot \frac{1}{x+1}$$

$$\lim_{x \rightarrow -1} \frac{2(x+1)}{x+2} \cdot \frac{1}{x+1} = \frac{2}{-1+2}$$

$$= \frac{2}{1}$$

$$= 2$$