Name Key Date________ Class________

Day #14 Homework

For exercises 1 – 12, find the derivative of each function. Leave your answers with no negative or rational exponents and as single rational functions, when applicable.

1. $f(x)=5-2x^2-3x^3$ $f'(x)=-4x-9x^2$	2. $h(x)=\dfrac{2x^3+3x^2-2x}{x}=2x^2+3x-2$ $h'(x)=4x+3$
3. $h(x)=\dfrac{3}{x^7}=3x^{-7}$ $h'(x)=-21x^{-8}=-\dfrac{21}{x^8}$	4. $g(x)=\dfrac{2x^5}{x^8}=2x^{-3}$ $g'(x)=-6x^{-4}=-\dfrac{6}{x^4}$
5. $f(\theta)=-3\theta^2-\cos\theta$ $f'(\theta)=-6\theta+\sin\theta$	6. $h(x)=\sqrt[3]{x^2}=x^{2/3}$ $h'(x)=\frac{2}{3}x^{-1/3}=\dfrac{2}{3x^{1/3}}$ $=\dfrac{2}{3\sqrt[3]{x}}$
7. $g(\theta)=\sqrt{\theta}+2\sin\theta=\theta^{1/2}+2\sin\theta$ $g'(\theta)=\frac{1}{2}\theta^{-1/2}+2\cos\theta$ $=\dfrac{1}{2\theta^{1/2}}+2\cos\theta$ $=\dfrac{1}{2\sqrt{\theta}}+2\cos\theta$ $=\dfrac{1+4\sqrt{\theta}\cos\theta}{2\sqrt{\theta}}$	8. $p(x)=-2x^{\frac{3}{2}}+\sqrt{x}=-2x^{3/2}+x^{1/2}$ $p'(x)=-3x^{1/2}+\frac{1}{2}x^{-1/2}$ $=-3\sqrt{x}+\dfrac{1}{2\sqrt{x}}$ $=\dfrac{-6x+1}{2\sqrt{x}}$

9. $g(x) = (x+3)(2x-1)^2$

$= (x+3)(4x^2-4x+1)$

$= 4x^3 - 4x^2 + x + 12x^2 - 12x + 3$

$= 4x^3 + 8x^2 - 11x + 3$

$g'(x) = 12x^2 + 16x - 11$

10. $h(x) = \dfrac{x^2+2x-2}{x^3} = x^{-1} + 2x^{-2} - 2x^{-3}$

$h'(x) = -x^{-2} - 4x^{-3} + 6x^{-4}$

$= -\dfrac{1}{x^2} - \dfrac{4}{x^3} + \dfrac{6}{x^4}$

$= \dfrac{x^2-4x+6}{x^4}$

11. $f(x) = \dfrac{3x}{\sqrt[3]{x}} = 3x \cdot x^{-1/3} = 3x^{2/3}$

$f'(x) = 2x^{-1/3}$

$= \dfrac{2}{\sqrt[3]{x}}$

12. $h(x) = 6\sqrt{x} - 3\cos x = 6x^{1/2} - 3\cos x$

$h'(x) = 3x^{-1/2} + 3\sin x$

$= \dfrac{3}{\sqrt{x}} + 3\sin x$

$= \dfrac{3 + 3\sqrt{x}\sin x}{\sqrt{x}}$

13. For what value(s) of x will the slope of the tangent line to the graph of $h(x) = 4\sqrt{x}$ be -2? Find the equation of the line tangent to $h(x)$ at this/these x – values. Show your work.

$h(x) = 4x^{1/2} \qquad h'(x) = 2x^{-1/2} = \dfrac{2}{\sqrt{x}} = -2$

$\dfrac{2}{\sqrt{x}} = -2 \Rightarrow (2)^2 = (-2\sqrt{x})^2 \Rightarrow \pm 4 = 2x \Rightarrow x = \pm 2 \qquad \boxed{x=2}$

$x \neq -2$

$h(2) = 4\sqrt{2}$ P.O.T. $= (2, 4\sqrt{2})$

S.O.T. $= h'(2) = \dfrac{2}{\sqrt{2}}$

EQ'N: $y - 4\sqrt{2} = \dfrac{2}{\sqrt{2}}(x-2)$

14. Find the equation of the line tangent to the graph of $g(x)=\dfrac{2}{\sqrt[4]{x^3}}$ when $x=1$.

$g(x)=2x^{-3/4}$

$g'(x)=\frac{-3}{2}x^{-7/4}=-\frac{3}{2\sqrt[4]{x^7}}$

$g(1)=\frac{2}{\sqrt[4]{1^3}}=\frac{2}{1}=2$

P.O.T. $(1,2)$

S.O.T $g'(1)=-\frac{3}{2\sqrt[4]{1^7}}=-\frac{3}{2}$

EQ: $y-2=-\frac{3}{2}(x-1)$

15. The line defined by the equation $\frac{1}{2}x+3=-2(y-3)$ is the line tangent to the graph of a function $f(x)$ when $x=a$. What is the value of $f'(a)$? Show your work and explain your reasoning.

$2\left(\frac{1}{2}x+3=-2(y-3)\right)$

$x+6=-4(y-3)$

$x+6=-4y+12$

$4y=-x+6 \rightarrow y=\frac{-1}{4}x+\frac{3}{2}$

~~SLOPE~~ $f'(a)$ = SLOPE OF TANGENT AT $x=a$, WHICH IS $-\frac{1}{4}$

16. The line defined by the equation $y-3=-\frac{2}{3}(x+3)$ is the line tangent to the graph of a function $f(x)$ at the point $(-3, 3)$. What is the equation of the normal line when $x=-3$. Explain your reasoning.

SLOPE OF NORMAL LINE = $\frac{3}{2}$ (OPP. RECIP.)

EQ: $y-3=\frac{3}{2}(x+3)$

NORMAL LINE HAS THE SAME $(x, f(x))$ AS TANGENT LINE

17. Determine the value(s) of x at which the function $f(x)=x^4-8x^2+2$ has a horizontal tangent.

$f'(x)=0=4x^3-16x$

$4x^3-16x=0$

$4x(x^2-4)=0$

$4x=0 \qquad x^2-4=0$

$x=0,\ x=\pm 2$

18. Determine the value(s) of θ at which the function $f(\theta)=\sqrt{3}\theta+2\cos\theta$ has a horizontal tangent on

the interval $[0, 2\pi)$. $f(\theta) = \sqrt{3}\,\theta + 2\cos\theta$

$f'(\theta) = \sqrt{3} - 2\sin\theta = 0$

$-2\sin\theta = -\sqrt{3}$

$\sin\theta - \frac{3}{\sqrt{2}}$ $\quad \theta = \frac{\pi}{6}, \frac{5\pi}{4}$

19. For what value(s) of k is the line $y = 4x - 9$ tangent to the graph of $f(x) = x^2 - kx$?

$= \cancel{f(x)})$

$f'(k) = 4$

$f'(x) = 2x - k = 4$

$2x - k = 4$

$2x - 4 = k$

$f(x) = x^2 - (2x-4)x$

$= x^2 - 2x^2 + 4x$

$= -x^2 + 4x$

Connections between $F(x)$ and $F'(x)$ for Polynomial and Trigonometric Functions