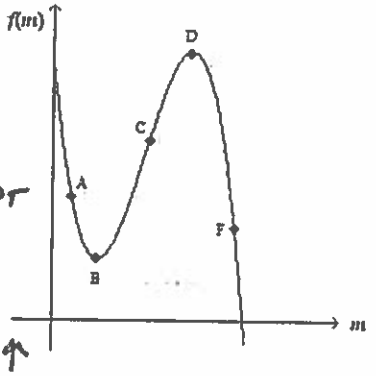
Name_____ Date_____ Class_____

Day #35 Homework

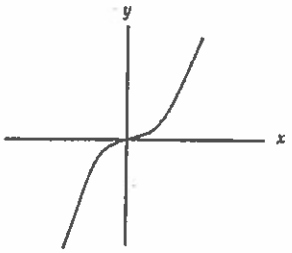
1. Pictured below is a function, $f(m)$. Complete the chart below indicating the sign (+ or – or 0) for $f(m)$, $f'(m)$ and $f''(m)$ at each of the indicated points.

Point	$f(m)$	$f'(m)$	$f''(m)$	
A	+	– (f dec.)	+	conc ↑
B	+	0 (horiz tan)	+	conc ↑
C	+	+ (f inc)	0	inflection pt
D	+	0 (horiz tan)	–	conc ↓
F	+	– (f dec)	–	conc ↓



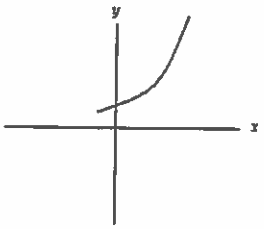
2. If, for all real numbers x, $f'(x) < 0$ and $f''(x) > 0$, which of the following curves could be part of the graph of $f(x)$? Explain your reasoning FOR EACH GRAPH.

Graph A



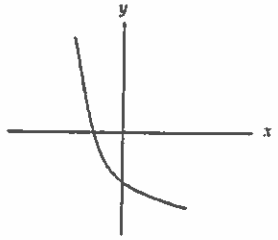
NO: f(x) IS NEVER DECREASING, AND IS ONLY CONCAVE UP ON (0,∞)

Graph B



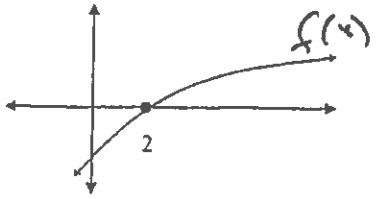
NO: f(x) IS NEVER DECREASING . . . IT IS, HOWEVER CONCAVE UP

Graph C



YES! f(x) IS DECREASING, SO f'(x) < 0 AND f(x) IS CONCAVE UP, SO f''(x) > 0

3. The graph of a twice differentiable function is shown below. Order the values of $f(2)$, $f'(2)$ and $f''(2)$ in order from least to greatest. Explain your reasoning.



f(2) = 0 : f WHEN x = 2, f(x) = 0

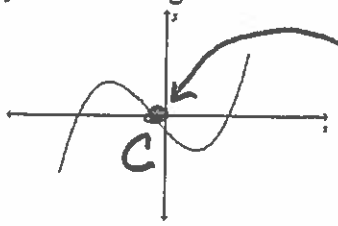
f'(2) > 0 : f IS INCREASING AT x = 2 (POSITIVE TANGENT SLOPE)

f''(2) < 0 : f IS CONCAVE DOWN

f''(2) < f(2) < f'(2)

The graph of $f'(x)$, the derivative of $f(x)$ is shown in each of the following questions. Answer the questions 4 – 6 using this graph.

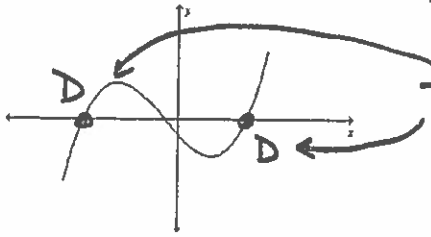
4. How many relative maximums does $f(x)$ have? Label these x values with the letter C. Explain your reasoning.



$f'(x)$ changes + to −

ONE RELATIVE MAX FOR $f(x)$

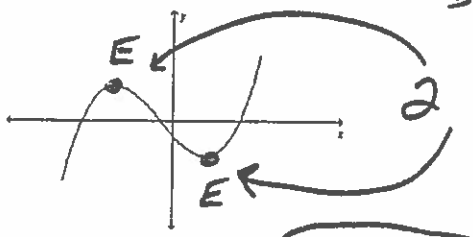
5. How many relative minimums does $f(x)$ have? Label these x values with the letter D. Explain your reasoning.



$f'(x)$ CHANGES − TO +

TWO RELATIVE MINIMA FOR $f(x)$

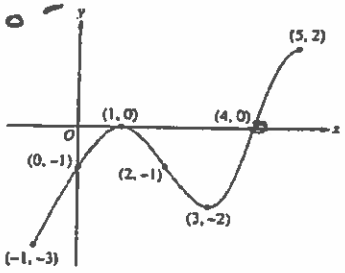
6. How many points of inflection does the graph of $f(x)$ have? Label these x values with the letter E. Explain your reasoning.



$f'(x)$ HAS A MAX OR MIN

2 INFLECTION POINTS ON $f(x)$

Pictured to the right is the graph of $f'(x)$. Use the graph to answer questions 7 – 13.



7. What are the value(s) of x where $f(x)$ has a relative maximum? Explain your reasoning.

f' + TO −

NONE: $f'(x)$ NEVER CHANGES FROM POSITIVE TO NEGATIVE

8. What are the value(s) of x where $f(x)$ has a relative minimum? Explain your reasoning.

f' − TO +

AT $x=4$ $f(x)$ HAS A MINIMUM: f' CHANGES SIGN FROM NEGATIVE TO POSITIVE

9. On what interval(s) is the graph of $f(x)$ increasing? Explain your reasoning.

$f(x)$ IS INCREASING ON $(4,5)$ WHERE $f'(x)$ IS POSITIVE

10. At what value(s) of x does the graph of $f(x)$ have a point of inflection? Explain your reasoning.

$f(x)$ HAS POINTS OF INFLECTION AT $x=1$ AND $x=3$, WHERE $f'(x)$ HAS A MAX OR MIN

11. On what interval(s) is the graph of $f(x)$ concave up or concave down? Explain your reasoning.

$f(x)$ IS CONCAVE UP ON $(-1,0)$ AND $(3,5)$ WHERE f'' IS POSITIVE (POSITIVE SLOPE OF TAN TO f')

$f(x)$ IS CONCAVE DOWN ON $(1,3)$ WHERE f'' IS NEGATIVE (NEGATIVE SLOPE OF TAN TO f')

12. If $f(2) = 4$, what is the equation of the normal line to the graph of $f(x)$ when $x = 2$?

AT $x=2$, $f'(2) = -1$ (SLOPE OF TANGENT)

SO SLOPE OF NORMAL $= 1$ SO: $y-4 = 1(x-2)$

13. If $f(2) = 4$, what is the tangent line approximation of $f(1.9)$? Is this an over or under approximation of $f(1.9)$? Explain your reasoning.

SINCE $f(x)$ IS CONCAVE DOWN AT $x=2$,

TANGENT EQ'N: $y-4 = -1(x-2)$ THIS IS AN OVER-APPROXIMATION

$y-4 = -1(1.9-2) \rightarrow y-4 = -1(-0.1) \rightarrow y-4 = 0.1 \rightarrow y = 4.1$

A function, F, is continuous on its domain of $[-2, 4]$. Additionally, $F(-2) = 5$, $F(4) = 1$ with F' and F'' have the properties shown in the table below. Use this information to answer questions 14 – 17.

$F(-2)=5$ $F(4)=1$

x	$-2 < x < 0$	$x = 0$	$0 < x < 2$	$x = 2$	$2 < x < 4$
$F'(x)$	Positive	Does not exist	Negative	0	Negative
$F''(x)$	Positive	Does not Exist	Positive	0	Negative

14. At what value(s) of x does F have relative extrema? Classify the extrema by type and give a reason for your answer.

CHANGE OF SIGN IN F'

AT $x=0$, $F(x)$ HAS A RELATIVE MAXIMUM: F' CHANGES + TO −

15. At what value(s) of x does F have a point of inflection? Justify your answer.

$f'=0$ AND f'' CHANGES SIGN

AT $x=2$, $F(x)$ HAS AN INFLECTION POINT: F'' CHANGES + TO − (CONCAVITY CHANGE.)

16. On what interval(s) is the graph of F increasing, decreasing, concave up or concave down? Justify your reasoning.

F IS INCREASING ON $(-2, 0)$ WHERE F' IS POSITIVE

F IS DECREASING ON $(0,2) \cup (2,4)$: $F' < 0$

F IS CONCAVE UP ON $(-2, 0) \cup (0, 2)$: $F'' > 0$

F IS CONCAVE DOWN ON $(2,4)$: $F'' < 0$

17. Suppose the equation of the tangent line drawn to F at $x = 2$ were used to evaluate $F(1.6)$ and $F(2.4)$. Would the approximations be under or over approximations? Justify your answers.

INFLECTION POINT AT $x=2$; ✓ $F''=0$

$1.6 < 2$, SO F IS CONCAVE UP: UNDER-APPROXIMATION

$2.4 > 2$: F IS CONCAVE DOWN: OVER-APPROXIMATION

↑ $F'' < 0$