Name KEY Date____________ Class__________

Day #16 Homework

1. If $g'(x) = (x-3)^2(x+1)$, determine on what intervals the graph of $g(x)$ is increasing or decreasing and identify the value(s) of x at which $g(x)$ has a relative maximum or minimum. Justify your reasoning and show your work.

CRITICAL VALUES:
$x = 3$, $x = -1$

-2 0 4

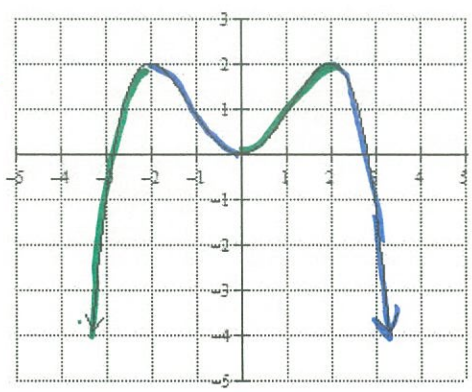
$g'(x)$ SIGN: $(+)(-)$ NEG | -1 | $(+)(+)$ POS | 3 | $(+)(+)$ POS

$g(x)$ IS INCREASING ON $(-1, \infty)$ WHERE $g'(x) > 0$

$g(x)$ IS DECREASING ON $(-\infty, -1)$ WHERE $g'(x) < 0$

$g(x)$ HAS A MINIMUM VALUE AT $x = -1$ WHERE $g'(x)$ CHANGES FROM ~~NEG. TO~~ NEG. TO POS. NO RELATIVE MAX

For exercises 2 – 4, use the graph of t function, $h(x)$, pictured to the right. Use the graph to identify the following. **Provide written justification.**



2. On what interval(s) is $h'(x) < 0$?

$h'(x) < 0$ ON $(-2, 0) \cup (2, \infty)$ WHERE $h(x)$ IS DECREASING (SLOPES OF TANGENT LINES ARE NEGATIVE)

3. On what interval(s) is $h'(x) > 0$?

$h'(x) > 0$ ON $(-\infty, -2) \cup (0, 2)$ WHERE $h(x)$ IS INCREASING (SLOPES OF TANGENT LINES ARE POSITIVE)

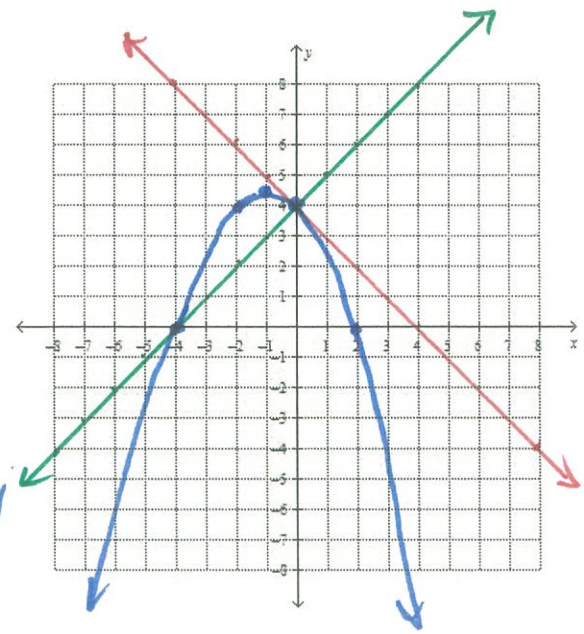
4. At what value(s) of x does $h'(x)$ change from positive to negative? From negative to positive?

$h'(x)$ CHANGES FROM POSITIVE TO NEGATIVE AT $x = -2$ AND $x = 2$, THE RELATIVE MAXIMA OF $h(x)$

$h'(x)$ CHANGES FROM NEGATIVE TO POSITIVE AT $x = 0$, THE RELATIVE MINIMUM OF $h(x)$

$(0,4)$

Consider the quadratic function $f(x) = -\frac{1}{2}x^2 - x + 4$.



5. Sketch an accurate graph of the function.

$f(2) = -\frac{1}{2}(4) - 2 + 4 = -2 - 2 + 4 = 0 \quad (2,0)$

6. Find $f'(x)$ and use it to find the absolute maximum of the graph of $f(x)$. (VERTEX)

$f'(x) = -x - 1$

MAX WHEN $f'(x) = 0$

$-x - 1 = 0$

$-x = 1$

$x = -1$

$f(-1) = -\frac{1}{2}(-1)^2 - (-1) + 4$

$= -\frac{1}{2} + 1 + 4$

$= 4.5$

$(-1, 4.5)$

7. Estimate the value of $f'(0)$ and explain what this value represents in terms of the graph of $f(x)$.

$f'(0) = -(0) - 1$

$= -1$

$f'(0)$ IS THE SLOPE OF THE TANGENT LINE TO THE GRAPH OF $f(x)$ AT $x = 0$

8. Find the equation of the tangent line to the graph of $f(x)$ at $x = 0$. Draw a graph of this line.

P.O.T: $(0,4)$

S.O.T: -1

$y - 4 = 7(x - 0$

$y - 4 = -x$

$y = -x + 4$

9. Sketch a graph of the normal line to the tangent line at $x = 0$. What is the equation of this line?

SLOPE OF NORMAL LINE IS OPPOSITE RECIPROCAL OF TANGENT LINE SLOPE, SO: 1

$y = x + 4$

10. Use the equation of the tangent line to approximate $f(0.1)$. Then, find $f(0.1)$ using the equation of $f(x)$. Is the approximation an under or over approximation of the actual value of $f(0.1)$? Based on the graph of $f(x)$, why do you suppose this is true?

$y - 4 = -1(0.1 - 0)$

$y = -0.1 + 4 = 3.9$

$f(0.1) = -\frac{1}{2}(0.1)^2 - 0.1 + 4$

$= -0.5(0.01) - 0.1 + 4$

$= -0.005 - 0.1 + 4$

$= 3.905$

APPROXIMATION IS OVER: TANGENT LINE IS LESS "STEEP", SO y DECREASES SLOWER

$$\rightarrow \frac{f(x+h)-f(x)}{h}$$

11. For what function does $\lim_{h\to 0}\frac{2\sin(x+h)-2\sin x}{h}$ give the derivative? Find the limit.

$f(x)=2\sin x$ THE LIMIT WILL BE THE DERIVATIVE OF $2\sin x$:

$$f'(x)=\frac{d}{dx}2\cdot\frac{d}{dx}\sin x=\boxed{2\cdot\cos x}$$

12. Find $\lim_{h\to 0}\frac{(x+h)^5-x^5}{h}$.

$f(x)=x^5$

$f'(x)=5x^4$

13. Find $\lim_{h\to 0}\frac{\sqrt{x}-\sqrt{x+h}}{h}$.

$$=\lim_{h\to 0}\frac{-\sqrt{x+h}-(-\sqrt{x})}{h}$$

SO: $f(x)=-\sqrt{x}=-x^{1/2}$

$$f'(x)=-\frac{1}{2}x^{-1/2}=-\frac{1}{2x^{1/2}}$$

$$=-\frac{1}{2\sqrt{x}}$$

14. If $f(x)=\frac{3x}{\sqrt{x}}$, what is the slope of the normal line to the graph of $f(x)$ when $x=4$?

$f(x)=3x\cdot x^{-1/2}=3x^{1/2}$

$f'(x)=\frac{3}{2}x^{-1/2}=\frac{3}{2\sqrt{x}}$

SLOPE OF TANGENT AT $x=4$ IS: $f'(4)=\frac{3}{2\sqrt{4}}=\frac{3}{2\cdot 2}=\frac{3}{4}$

SLOPE OF NORMAL LINE IS THE OPPOSITE RECIPROCAL OF $3/4$, OR $-\frac{4}{3}$

15. If $2x-3=5(y+1)$ is the equation of the normal line to the graph of $f(x)$ when $x=a$, find the value of $f'(a)$. Show your work and explain your reasoning.

$2x-3=5y+1$

$2x-4=5y$

$\frac{2}{5}x-\frac{4}{5}=y$

SLOPE OF ~~TANGE~~ NORMAL LINE IS $2/5$

$f'(a)$ IS THE SLOPE OF THE TANGENT LINE (OPP. RECIP. OF NORMAL LINE SLOPE), SO

$$-\frac{5}{2}$$

16. On the interval $[0, 2\pi)$, find the coordinates of the relative minimum(s) of $f(\theta) = \sqrt{3}\theta - 2\sin\theta$.

$f'(\theta) = \sqrt{3} - 2\cos\theta = 0$

$\sqrt{3} - 2\cos\theta = 0$

$-2\cos\theta = -\sqrt{3}$

$\cos\theta = \frac{\sqrt{3}}{2}$

$\theta = \frac{\pi}{6}, \frac{11\pi}{6}$

$0 \qquad \frac{\pi}{6} \qquad \pi \qquad \frac{11\pi}{6} \qquad 2\pi$

$\sqrt{3} - 2\cos(0)$
$\sqrt{3} - 2(1)$
$1.73 - 2$
NEG

$\sqrt{3} - (2\cos\pi)$
$-\sqrt{3} - 2(-1)$
POS

$\sqrt{3} - 2\cos 2\pi$
$\sqrt{3} - 2(1)$
NEG

MINIMUM

$f\left(\frac{\pi}{6}\right) = \sqrt{3}\left(\frac{\pi}{6}\right) - 2\sin\left(\frac{\pi}{6}\right)$

$= \frac{\pi\sqrt{3}}{6} - 2\left(\frac{1}{2}\right)$

$= \frac{\pi\sqrt{3}}{6} - 1$

$\left(\frac{\pi}{6}, \frac{\pi\sqrt{3}}{6} - 1\right)$

The derivative of a function $f(x)$ is $f'(x) = (3-x)^2(x+5)$. Use this derivative for exercises 17 and 18.

17. At what value(s) of x does the graph of $f(x)$ have a relative maximum? Justify your answer.

$f'(x) = 0$ AT $x = 3$ & $x = -5$ (CRITICAL VALUES)

$-6 \qquad -5 \qquad 0 \qquad 3 \qquad 4$

(+)(−) NEG/ DECR.

(+)(+) POS/ INC

(+)(+) POS/ INC

$f(x)$ HAS NO RELATIVE MAXIMUM, SINCE $f'(x)$ NEVER CHANGES SIGN FROM POSITIVE TO NEGATIVE

18. Use the equation of the tangent line to approximate the value of $f(2.1)$ if $f(2) = -3$.

P.O.T: $(2, -3)$

S.O.T: $f'(2) = (3-2)^2(2+5)$

$= 1^2(7)$

$= 7$

$\boxed{y + 3 = 7(x-2)}$

IF $x = 2.1$:

$y + 3 = 7(2.1 - 2)$

$y = 7(0.1) - 3$

$= 0.7 - 3$

$= -2.3$