The Law of Sines

Students will utilize the Law of Sines to find the missing sides and angles of acute and obtuse triangles.

What is the Law of Sines?

**Law of Sines**

Used to find the missing sides and angles of oblique (non-right) triangles. Right triangle trigonometry won’t work.

\[
\begin{align*}
\sin B &= \frac{h}{a} \\
\sin A &= \frac{b}{b}
\end{align*}
\]

\[
\begin{align*}
a \cdot \sin B &= b \cdot \sin A \\
\frac{a}{\sin A} &= \frac{b}{\sin B}
\end{align*}
\]

In any \( \triangle ABC \) with angles \( A, B, \) and \( C \) and opposite sides \( a, b, \) and \( c \) the following equation is true:

\[
\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}
\]

The Law of Sines can be used to solve the following cases of oblique triangles:

1. Angle-Angle-Side: \( \text{AAS} \)
2. Angle-Side-Angle: \( \text{ASA} \) (side is included)
3. Side-Side-Angle: \( \text{SSA} \) (special case)

Law of sines can be used for both acute and obtuse triangles.
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Example 1: AAS

Solve the ΔABC if \( m \angle A = 46^\circ, m \angle C = 63^\circ, c = 56 \)

a. Sketch the triangle

![Triangle diagram]

b. Find the measure of angle B.

\[ B = 180^\circ - 46^\circ - 63^\circ = 71^\circ \]

c. Solve for side a and side b.

\[
\frac{\text{SIDE } a}{\sin A} = \frac{c}{\sin C} \quad \frac{\text{SIDE } b}{\sin B} = \frac{c}{\sin C}
\]

\[
\frac{a}{\sin 46^\circ} = \frac{56}{\sin 63^\circ} \quad \frac{b}{\sin 71^\circ} = \frac{56}{\sin 63^\circ}
\]

\[
a = \frac{56 \sin 46^\circ}{\sin 63^\circ} \quad b = \frac{56 \sin 71^\circ}{\sin 63^\circ}
\]

\[
a \approx 45.21 \quad b \approx 59.43
\]
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#### Example 2: ASA (side included)

Solve \( \triangle ABC \) if \( m \angle A = 40^\circ \), \( m \angle C = 125^\circ \), \( b. = 12 \)

**a. Sketch the triangle**

![Sketch of \( \triangle ABC \) with given angles and side](image)

**b. Find the measure of angle B.**

\[
B = 180^\circ - 125^\circ - 40^\circ = 15^\circ
\]

**c. Solve for side \( a \) and side \( c \).**

\[
\frac{c}{\sin 125^\circ} = \frac{12}{\sin 40^\circ} \quad \frac{a}{\sin 40^\circ} = \frac{12}{\sin 15^\circ}
\]

\[
c = \frac{12 \sin 125^\circ}{\sin 15^\circ} \approx 37.98
\]

\[
a = \frac{12 \sin 40^\circ}{\sin 15^\circ} \approx 29.8
\]
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Example 3: SSA ("the ambiguous case")

Solve \( \triangle ABC \) if \( \angle A = 43^\circ \), \( a = 81 \), \( b = 62 \)

\[ a. \text{ Sketch the triangle.} \]

\[ \begin{align*}
\angle A &= 43^\circ \\
\angle B &= \sin^{-1}\left(\frac{62 \sin 43^\circ}{81}\right) \approx 31.5^\circ \\
\angle C &= 180^\circ - 43^\circ - 31.5^\circ = 105.5^\circ
\end{align*} \]

\[ b. \text{ Find the measure of angle } B \text{ and determine the number of solutions.} \]

\[ \begin{align*}
\frac{\sin B}{62} &= \frac{\sin 43^\circ}{81} \\
B &= \sin^{-1}\left(\frac{62 \sin 43^\circ}{81}\right) \approx 31.5^\circ
\end{align*} \]

\[ \frac{c}{\sin 105.5^\circ} = \frac{81 \sin 105.5^\circ}{\sin 43^\circ} \]

\[ c \approx 114.4 \]

\[ c. \text{ Solve for side } c. \]
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**Example 4: SSA (“the ambiguous case“)**

Solve $\triangle ABC$ if $m\angle A = 50^\circ$, $a = 10$, $b = 20$

*a. Sketch the triangle*

![Diagram of triangle ABC with labels a=10, b=20, and angle A=50°.]

*b. Find the measure of angle B and determine the number of solutions.*

\[
\frac{\sin B}{20} = \frac{\sin 50^\circ}{10}
\]

\[
B = \sin^{-1}\left(\frac{20 \sin 50^\circ}{10}\right) = \text{UNDEF}
\]

**NO Δ EXISTS**

*c. Solve for side c.*
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Example 5: SSA (“the ambiguous case“)

Solve ΔABC if m∠A = 35°, a = 12, b = 16

a. Sketch the triangle.

b. Find the measure of angle B and determine the number of solutions.

\[
\frac{\sin B}{16} = \frac{12}{\sin 35°} \\
B = \sin^{-1}\left(\frac{16 \sin 35°}{12}\right) \approx 50° \quad B_2 = 130°
\]

C_1 = 180° - 50° - 35° = 95° \quad C_2 = 180° - 30° - 35° = 15°

C. Solve for side c.

\[
\frac{c_1}{\sin 95°} = \frac{12}{\sin 35°} \quad \frac{c_2}{\sin 15°} = \frac{12}{\sin 35°}
\]

C_1 \approx 20.8 \quad C_2 \approx 5.4
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**Example 6: Applications**

In a children’s amusement park, the administration wanted to build a walker’s bridge along a Pond in the midst of the park from point A to B. For this purpose an Engineer identified the approximate straight edge of the Pond near B and marked a point C at a distance 12 m from B. He was able to measure the angles ACB and ABC as 102° and 68° to estimate the length of the bridge to be built. Find the approximate length of the bridge rounded to a meter.
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Example 7: Applications

Height of Blimp In order to find the height of the Goodyear blimp, observers at A and B, 158 yards apart, measure the following angles: \( \alpha = 45.0^\circ \) and \( \beta = 60.0^\circ \). (See the diagram.) How high is the blimp?

\[
\begin{align*}
\sin 60^\circ &= \frac{h}{432} \\
432 \sin 60^\circ &= h \\
h &\approx 374 \text{ yd}
\end{align*}
\]

\[
\begin{align*}
\frac{x}{\sin 45^\circ} &= \frac{158}{\sin 15^\circ} \\
x &= \frac{158 \sin 45^\circ}{\sin 15^\circ} \approx 432 \text{ yd}
\end{align*}
\]
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#### Example 8: Applications

An antenna is to be placed on a hillside that has a slope of 15°. Guywires are to be placed 150 feet up on the tower. If the angle of elevation of the downhill wire is 38° and the angle of elevation of the uphill wire is 48°, how long is each wire?

\[
\frac{x}{\sin 15°} = \frac{150}{\sin 38°} \quad \Rightarrow \quad x \approx 235.3'
\]

\[
\frac{y}{\sin 75°} = \frac{150}{\sin 48°} \quad \Rightarrow \quad y \approx 195'
\]