

Introduction to Rational Functions

Algebra 3-4

- Students will be able to simplify rational expressions.

What is a rational function?

If $N(x)$ and $D(x)$ are functions with $D(x) \neq 0$,

$$R(x) = \frac{N(x)}{D(x)} = \frac{a_n x^n + a_{n-1} x^{n-1} + \dots + a_1 x^1 + a_0}{b_n x^n + b_{n-1} x^{n-1} + \dots + b_1 x^1 + b_0} \text{ is a } \underline{\text{Rational Function}}$$

In "simpler terms", a rational function is a fraction that has polynomials for both the numerator and denominator.

Examples of Rational Functions:

$$f(x) = \frac{4x-1}{2x}; \quad h(x) = \frac{3}{6x+4}; \quad b(t) = \frac{5t^2-2t+1}{7}; \quad a(d) = \frac{4d^5-6d^4+9d^2-d+1}{d^7-7d^3+8d^2-2}$$

Since Rational Functions are just fractions, all "fraction rules" apply:

- Factors will reduce "top-to-bottom"
- Addition and subtraction of rational functions require a common denominator
- When you multiply rational functions, you multiply "numerator with numerator" and "denominator with denominator"
- When you divide one rational function by another, you multiply by the reciprocal of the divisor (what you're dividing by . . .)

Examples of simplification of Rational Expressions:

$$\frac{15x}{3} = 5x \qquad \frac{21x^2}{3x} = 7x \qquad \frac{6x+4}{2x} = \frac{\cancel{2}(3x+2)}{\cancel{2}x}$$

$$\frac{3x+21}{x^2+7x} = \frac{\cancel{3}(x+7)}{x\cancel{(x+7)}} = \frac{3}{x} \qquad \frac{(x-4)\cancel{(x+1)}}{x+1} = x-4$$

$$\frac{x^2+2x-8}{x^2-2x} = \frac{(x+4)\cancel{(x-2)}}{x\cancel{(x-2)}} = \frac{x+4}{x}$$

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How are rational expressions added or subtracted?

Addition and subtraction of rational expressions

$$\frac{4x}{3} + \frac{10x}{3} = \frac{14x}{3}$$

$$\frac{17a}{14} - \frac{a \cdot 2}{7 \cdot 2} = \frac{17a}{14} - \frac{2a}{14} = \frac{15a}{14}$$

$$\frac{3(v+2)}{3 \cdot 5} + \frac{v \cdot 5}{3 \cdot 5} = \frac{3v+6}{15} + \frac{5v}{15} = \frac{8v+6}{15}$$

$$\frac{5 \cdot 2m-1}{5 \cdot 3m} - \frac{8m \cdot m}{15 \cdot m} = \frac{10m-5}{15m} - \frac{8m^2}{15m}$$

$x^2 - 2x - x + 2 = (x-2)(x+2)$
 $x^2 + x - 4 = (x+1)(x-4)$
 $x^2 - 2x - x + 2 = (x-2)(x+2)$
 $x^2 + x - 4 = (x+1)(x-4)$
 $x^2 - 4 + x^2 - 1 = 2x^2 - 5$
 $x^2 - 3x + 2 = \frac{2x^2 - 5}{x^2 - 3x + 2}$

$$\frac{(10m-5) - 8m^2}{15m} = \frac{-8m^2 + 10m - 5}{15m}$$

Multiplication and division of rational expressions

$$\frac{x}{7} \cdot \frac{3x}{2} = \frac{3x^2}{14}$$

$$\frac{x+3}{2} \cdot \frac{x}{7} = \frac{x^2+3x}{14}$$

$$\frac{x-1}{x} \cdot \frac{x-4}{x+3} = \frac{x^2-5x+4}{x^2+3x}$$

$$\frac{4x}{13} \div \frac{12}{x} = \frac{4x}{13} \cdot \frac{x}{12} = \frac{x^2}{39}$$

$$\frac{(x+1)}{(x-7)} \div \frac{(x+1)}{(x+3)} = \frac{x+1}{x-7} \cdot \frac{x+3}{x+1} = \frac{x+3}{x-7}$$

$$\frac{x^2+3x}{x-1} \div \frac{x+3}{x+2} = \frac{x(x+3)}{x-1} \cdot \frac{x+2}{x+3} = \frac{x^2+2x}{x-1}$$

$$\frac{x(x+3)}{x-1} \cdot \frac{x+2}{x+3}$$

$$\frac{x^2+2x}{x-1}$$