



Honors PreCalculus

Sequences



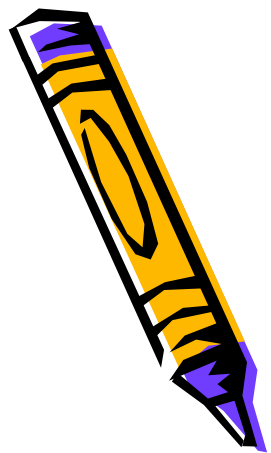
What's a sequence?

- An ordered progression of numbers

- Finite sequence: 2, 4, 6, 8, 10

- Infinite sequence: 1, 3, 9, . . . , 3^k , . . .

- Fibonacci sequence: 1, 1, 2, 3, 5, 8, 13, . . .



What's a sequence?

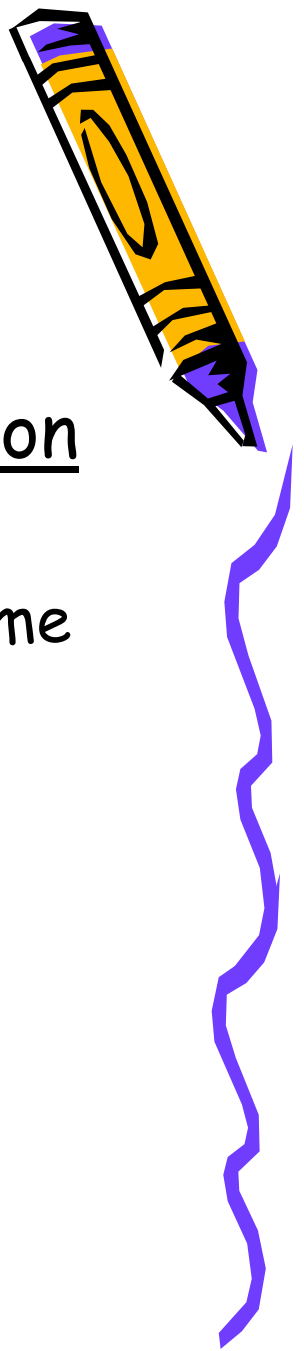
- Arithmetic sequence

- Numbers in the sequence have a common difference

- Add or subtract the same number each time

- General "rule"

- $a_n = a_1 + (n - 1)d$

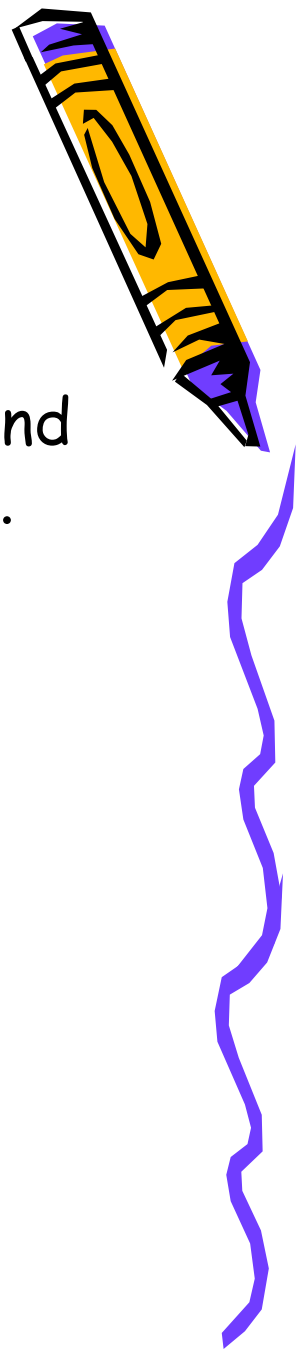


Arithmetic Sequences

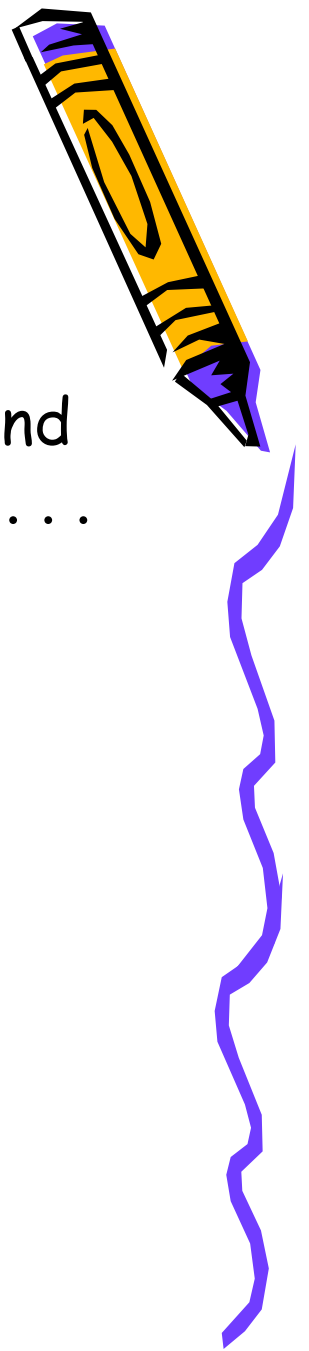
Example 1:

Find the common difference, the rule (equation) and the 10th term of the sequence: -3, -1, 1, 3, 5, ...

- Equation (rule): $a_n = a_1 + (n - 1)d$
 - $a_1 = -3$
 - $d = 2$
 - So: $a_n = -3 + (n - 1)(2) = -3 + 2n - 2 = 2n - 5$
 - 10th term: $n = 10$, so $a_{10} = 2(10) - 5 = 20 - 5 = 15$



Arithmetic Sequences



Example 2:

Find the common difference, the rule (equation) and the 10th term of the sequence: 6, 2, -2, -6, -10, ...

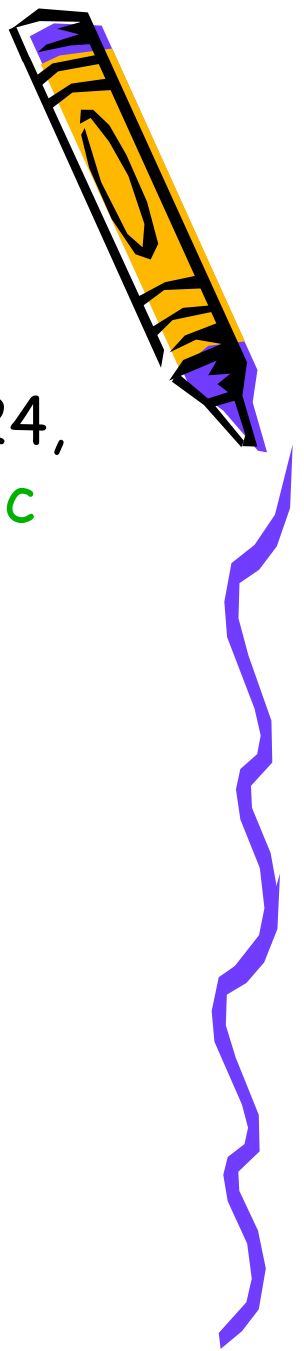
- Equation (rule): $a_n = a_1 + (n - 1)d$
 - $a_1 = 6$
 - $d = -4$
 - So: $a_n = 6 + (n - 1)(-4) = 6 - 4n + 4 = -4n + 10$
 - 10th term: $n = 10$, so $a_{10} = -4(10) + 10 = -40 + 10 = -30$



Constructing Sequences

Example 3:

If the 2nd and 5th terms of a sequence are 3 and 24, respectively, find the equation of the **arithmetic** sequence



What's a sequence?

- Geometric sequence

- Numbers in the sequence have a common ratio

- Multiply or divide by the same number each time

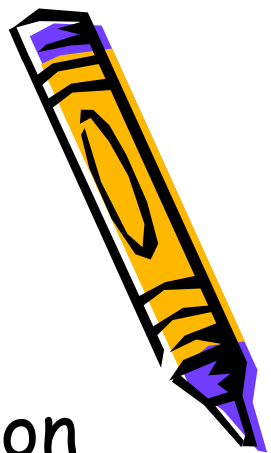
- General "rule"

- $a_n = a_1(r^{n-1})$

$\rightarrow a b^x$

\swarrow term

\swarrow $r =$ COMMON RATIO

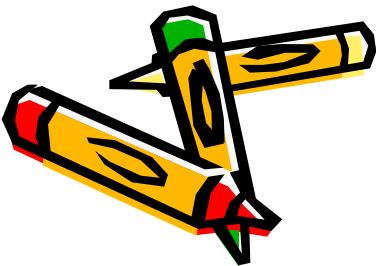


Geometric Sequences

Example 4:

Find the common ratio, the rule (equation) and the 10th term of the sequence: 3, 6, 12, 24, 48, ...

- Equation (rule): $a_n = a_1(r^{n-1})$
 - $a_1 = 3$
 - $r = 2$
 - So: $a_n = 3(2^{n-1})$
 - 10th term: $n = 10$, so $a_{10} = 3(2^{10-1}) = 3(2^9) = 1536$



Constructing Sequences



Example 5:

$$a_n = a_1(r)^{n-1}$$

$$a_1 = \frac{3}{2} = 1.5 \quad r = 2$$

$$a_n = 1.5(2)^{n-1}$$

If the 2nd and 5th terms of a sequence are 3 and 24, respectively, find the equation of the **geometric** sequence

$$a_2 = 3 = a_1(r)^{2-1} \Rightarrow 3 = a_1 r^1 \Rightarrow a_1 = \frac{3}{r}$$

$$a_5 = 24 = a_1(r)^{5-1} \Rightarrow 24 = a_1 r^4 \Rightarrow a_1 = \frac{24}{r^4}$$

$$\frac{3}{r} = \frac{24}{r^4} \Rightarrow \frac{3r^4}{r} = \frac{24r}{r} \Rightarrow 3r^3 = 24$$

$$r^3 = 8$$

$$r = 2$$

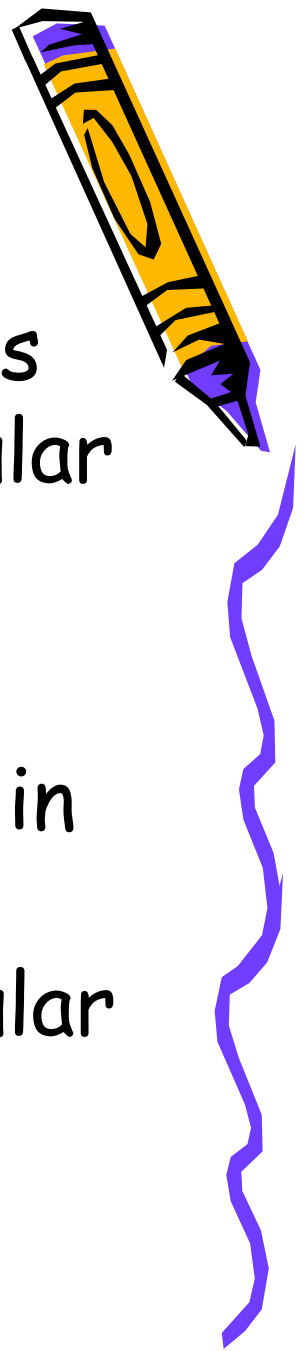
$$a_1 = 1.5$$



Limit of a sequence

A sequence converges if the numbers in the sequence approach a particular number

A sequence diverges if the numbers in the sequence approach infinity, or don't actually approach any particular number



Limit of a sequence



Examples: $1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \dots, \frac{1}{n}, \dots$ Converges to 0

$\frac{2}{1}, \frac{3}{2}, \frac{4}{3}, \frac{5}{4}, \dots, \frac{n+1}{n} = 1 + \frac{1}{n}$ Converges to 1

$2, 4, 6, 8, 10, \dots$ Diverges

$-1, 1, -1, 1, \dots, (-1)^n, \dots$ Diverges

