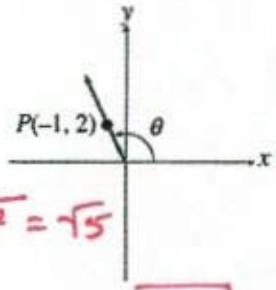
Trigonometric Functions of Any Angle

Name KEY

'valuate the six trigonometric functions of the angle θ. Leave your answers in **simplified fraction form.**

1.



$x = -1$
$y = 2$
$r = \sqrt{5}$

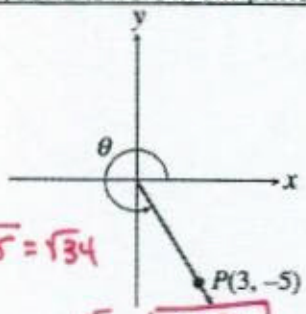
$r = \sqrt{(-1)^2 + 2^2} = \sqrt{5}$

$\sin\theta = \frac{2}{\sqrt{5}} = \boxed{\frac{2\sqrt{5}}{5}}$ $\csc\theta = \boxed{\frac{\sqrt{5}}{2}}$

$\cos\theta = \frac{-1}{\sqrt{5}} = \boxed{-\frac{\sqrt{5}}{5}}$ $\sec\theta = \boxed{-\sqrt{5}}$

$\tan\theta = \frac{2}{-1} = \boxed{-2}$ $\cot\theta = \boxed{-\frac{1}{2}}$

2.



$x = 3$
$y = -5$
$r = \sqrt{34}$

$\sqrt{9+25} = \sqrt{34}$

$\sin\theta = -\frac{5}{\sqrt{34}} = \boxed{-\frac{5\sqrt{34}}{34}}$ $\csc\theta = \boxed{-\frac{\sqrt{34}}{5}}$

$\cos\theta = \frac{3}{\sqrt{34}} = \boxed{\frac{3\sqrt{34}}{34}}$ $\sec\theta = \boxed{\frac{\sqrt{34}}{3}}$

$\tan\theta = \boxed{-\frac{5}{3}}$ $\cot\theta = \boxed{-\frac{3}{5}}$

In exercises 3 – 8, point P is on the terminal side of angle θ. Evaluate the indicated trigonometric functions for θ. If the function is undefined, write "undefined". Leave your answers in **simplified fraction form.**

3. $P(3, 4)$ $r = 5$

$\sin\theta = \boxed{\frac{4}{5}}$

$\cos\theta = \boxed{\frac{3}{5}}$

$\tan\theta = \boxed{\frac{4}{3}}$

4. $P(-4, -6)$ $r = \sqrt{16+36} = \sqrt{52} = 2\sqrt{13}$

$\csc\theta = -\frac{2\sqrt{13}}{6} = \boxed{-\frac{\sqrt{13}}{3}}$

sin $\frac{-6}{2\sqrt{13}}$

$\sec\theta = -\frac{2\sqrt{13}}{4} = \boxed{-\frac{\sqrt{13}}{2}}$

$\frac{-4}{2\sqrt{13}}$

$\cot\theta = \frac{4}{6} = \boxed{\frac{2}{3}}$

5. $P(0, 5)$ $r = 5$

$\sin\theta = \frac{5}{5} = \boxed{1}$

$\sec\theta = \frac{5}{0} \rightarrow$ UNDEFINED

cos: $\frac{0}{5}$

$\tan\theta = \frac{5}{0} =$ UNDEFINED

6. $P(-3, 0)$ $r = 3$

$\csc\theta = \frac{3}{0} \rightarrow$ UNDEFINED

sin: $\frac{0}{3}$

$\cos\theta = -\frac{3}{3} = \boxed{-1}$

$\cot\theta =$

7. $P(5, -2)$ $r = \sqrt{25+4} = \sqrt{29}$

$\sin\theta = -\frac{2}{\sqrt{29}} = \boxed{-\frac{2\sqrt{29}}{29}}$

$\cos\theta = \frac{5}{\sqrt{29}} = \boxed{\frac{5\sqrt{29}}{29}}$

$\cot\theta = \boxed{-\frac{5}{2}}$

8. $P(22, -22)$ $r = \sqrt{22^2 + 22^2} = 22\sqrt{2}$

$\cos\theta = \frac{22}{22\sqrt{2}} = \frac{1}{\sqrt{2}} = \boxed{\frac{\sqrt{2}}{2}}$

$\sec\theta = \frac{\sqrt{2}}{1} = \boxed{\sqrt{2}}$

$\tan\theta = -\frac{22}{22} = \boxed{-1}$

For problems 9 – 30, evaluate without using a calculator. Leave your answers in **simplified fraction form.**

9. $\cos 120° = \boxed{-\frac{1}{2}}$

10. $\tan 300° = \boxed{-\sqrt{3}}$

11. $\sec\frac{\pi}{3} = \boxed{2}$

12. $\csc\frac{3\pi}{4} = \boxed{\sqrt{2}}$

13. $\sin\frac{13\pi}{6} = \boxed{\frac{1}{2}}$

14. $\cos\frac{7\pi}{3} = \boxed{\frac{1}{2}}$

15. $\tan -\frac{15\pi}{4} = \boxed{1}$

16. $\cot\frac{13\pi}{4} = \boxed{1}$

17. $\cos\frac{23\pi}{6} = \boxed{\frac{\sqrt{3}}{2}}$

18. $\cos\frac{17\pi}{4} = \boxed{\frac{\sqrt{2}}{2}}$

19. $\sin\frac{11\pi}{3} = \boxed{-\frac{\sqrt{3}}{2}}$

21. $\cot\frac{19\pi}{6} = \boxed{\sqrt{3}}$

22. $\sin(-450°) = \boxed{-1}$

23. $\tan(-270°) = \boxed{\text{UNDEFINED}}$

$(0,1)$ $\frac{1}{0}$

24. $\cos -\frac{7\pi}{2} = \boxed{0}$

$(0,1)$

25. Find $\sin\theta$ and $\tan\theta$ if $\cos\theta = \frac{2}{3}$ and $\cot\theta > 0$.

$|x| = 2$
$r = 3$
$2^2 + y^2 = 3^2$
$y^2 = 5$ $|y| = \sqrt{5}$

$\sin\theta = \frac{\sqrt{5}}{3}$

$\tan\theta = \frac{\sqrt{5}}{2}$

26. Find $\cos\theta$ and $\cot\theta$ if $\sin\theta = \frac{1}{4}$ and $\tan\theta < 0$.

$|y| = 1$
$r = 4$
$|x| = \sqrt{15}$

$\cos\theta = -\frac{\sqrt{15}}{4}$

$\cot = -\frac{\sqrt{15}}{1} = -\sqrt{15}$

$\frac{x}{y}$

$x^2 + 1 = 16$ $x^2 = 15$ $x = \sqrt{15}$

27. Find $\tan\theta$ and $\sec\theta$ if $\sin\theta = -\frac{2}{5}$ and $\cos\theta > 0$.

$|y| = 2$
$r = 5$
$|x| = \sqrt{21}$

$\tan\theta = -\frac{2}{\sqrt{21}} = \frac{-2\sqrt{21}}{21}$

$\sec\theta = \frac{5}{\sqrt{21}} = \frac{5\sqrt{21}}{21}$

r/x

$x^2 + 4 = 25$ $x^2 = 21$ $x = \sqrt{21}$

28. Find $\sin\theta$ and $\cos\theta$ if $\cot\theta = \frac{3}{7}$ and $\sec\theta < 0$.

$|x| = 3$
$|y| = 7$
$r = \sqrt{58}$

$\sin\theta = -\frac{7}{\sqrt{58}} = -\frac{7\sqrt{58}}{58}$

$\cos\theta = -\frac{3}{\sqrt{58}} = \frac{-3\sqrt{58}}{58}$

$\sqrt{49+9} = r = \sqrt{58}$

29. Find $\sec\theta$ and $\csc\theta$ if $\cot\theta = -\frac{4}{3}$ and $\cos\theta < 0$.

$|x| = 4$
$|y| = 3$
$r = 5$

$\sec\theta = -\frac{5}{4}$

$\csc\theta = \frac{5}{3}$

$r = \sqrt{16+9} = 5$

30. Find $\csc\theta$ and $\cot\theta$ if $\tan\theta = -\frac{4}{3}$ and $\sin\theta > 0$.

$|x| = 3$
$|y| = 4$
$r = 5$

$\csc\theta = \frac{5}{4}$

$\cot\theta = -\frac{3}{4}$

$r = \sqrt{9+16} = 5$